1. The function $g$ is related to one of the parent functions discussed in class. (8 points)

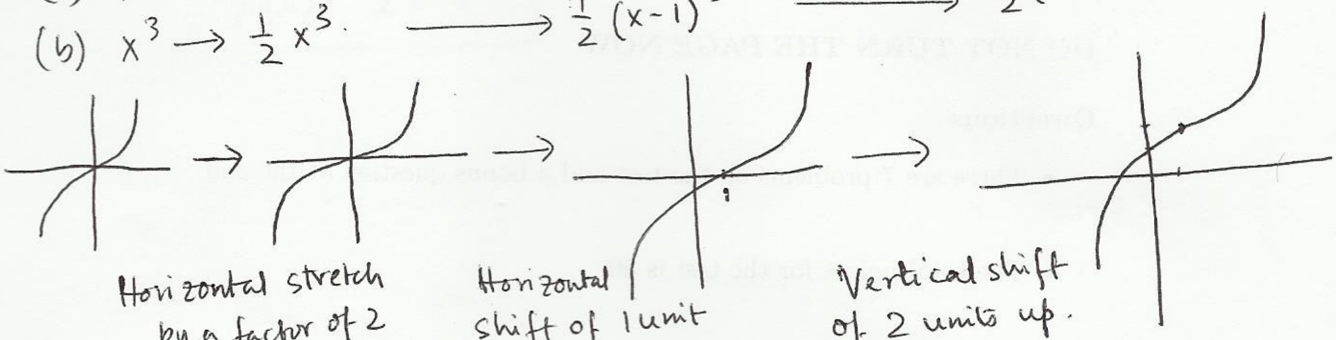
   (a) Identify the parent function $f$.

   (b) Describe the sequence of transformations from $f$ to $g$ and sketch the graph of $g$. Show your work.

$$g(x) = \frac{1}{2}(x-1)^3 + 2$$

(a) $f(x) = x^3$

(b) $x^3 \rightarrow \frac{1}{2}x^3 \longrightarrow \frac{1}{2}(x-1)^3 \longrightarrow \frac{1}{2}(x-1)^3 + 2$



Horizontal stretch by a factor of 2

Horizontal shift of 1unit to the right

Vertical shift of 2 units up.

2. Consider the following function. (7 points)

$$f(x) = \frac{x-8}{x}$$

   (a) Find the inverse function of $f$. Show your work.

   (b) State the domain and range of $f$ and $f^{-1}$. Show your work.

(a) $y = \frac{x-8}{x}$

$x = \frac{y-8}{y}$

$xy = y - 8$

$xy - y = -8$

$y(x-1) = -8$

$y = \frac{-8}{x-1}$

$\boxed{f^{-1}(x) = \frac{-8}{x-1}}$

(b) Domain $(f) = (-\infty, 0) \cup (0, \infty)$

Domain $(f^{-1}) = (-\infty, 1) \cup (1, \infty)$

Range $(f) = (-\infty, 1) \cup (1, \infty)$

Range $(f^{-1}) = (-\infty, 0) \cup (0, \infty)$

3. Identify the vertex, axis of symmetry, and $x$-intercepts and then sketch the graph of $f$. Show your work. (8 points)

$$f(x) = 2x^2 + 8x - 7$$

x-int

$2x^2+8x-7=0$

$x = \frac{-8 \pm \sqrt{64+56}}{4}$

$= \frac{-8 \pm \sqrt{120}}{4}$

$= \frac{-8 \pm 2\sqrt{30}}{4}$

$= -2 \pm \frac{\sqrt{30}}{2}$

$= \boxed{0.74, -4.74}$

$f(x) = 2(x^2+4x) - 7$

$= 2(x^2+4x+4-4) - 7$

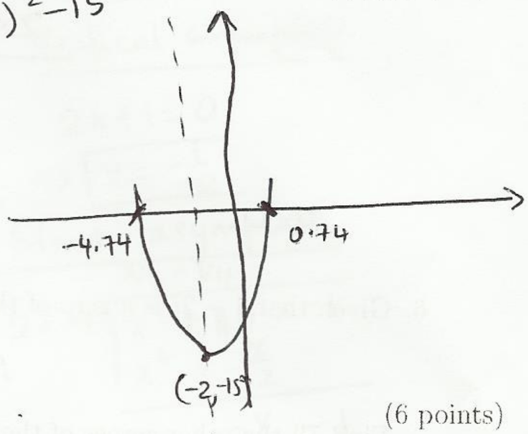
$= 2((x+2)^2-4) - 7$

$= 2(x+2)^2 - 8 - 7$

$= 2(x+2)^2 - 15$

$\therefore$ Vertex: $(-2,-15)$

Axis: $x=-2$




4. Consider the following function. (6 points)

$$f(x) = x^3 - 4x^2 - 9x + 36$$

(a) Find all the real zeroes of the function $f$.

(b) Determine the multiplicity of each zero and mention whether the graph of $f$ touches or crosses the $x$-axis at that point.

(a) $x^3-4x^2-9x+36=0$

$\Rightarrow x^2(x-4) - 9(x-4) = 0$

$\Rightarrow (x^2-9)(x-4) = 0$

$\Rightarrow (x-3)(x+3)(x-4) = 0$

$\Rightarrow \boxed{x = 3,-3,4}$

(b) 3, −3, 4 all have multiplicity 1 therefore crosses the x-axis at these points.

5. Use long division to divide. Find the quotient and remainder. Show your work. (5 points)

$$(2x^4 - 4x^3 + 8x - 5) \div (x^2 - 1)$$

$$
\begin{array}{r|l}
 & 2x^2 - 4x + 2 \\
\hline
x^2 - 1 & 2x^4 - 4x^3 + 0 \cdot x^2 + 8x - 5 \\
 & 2x^4 \qquad - 2x^2 \\
\hline
 & \quad -4x^3 + 2x^2 + 8x - 5 \\
 & \quad -4x^3 \qquad + 4x \\
\hline
 & \qquad 2x^2 + 4x - 5 \\
 & \qquad 2x^2 \qquad - 2 \\
\hline
 & \qquad\qquad 4x - 3
\end{array}
$$

$$\boxed{\begin{array}{l} \text{Quotient} = 2x^2 - 4x + 2 \\ \text{Remainder} = 4x - 3 \end{array}}$$

6. Given that $1 - 2i$ is a zero of the function

$$f(x) = x^3 - 3x^2 + 7x - 5$$

Find all the other zeroes of the function. Show your work. (8 points)

$1 + 2i$ is also a zero

$\therefore$ $(x - (1-2i))(x - (1+2i))$ are factors of $f(x)$

$\Rightarrow$ $(x - 1 + 2i)(x - 1 - 2i)$ is a factor of $f(x)$

$\Rightarrow$ $(x-1)^2 - (2i)^2 = x^2 - 2x + 1 + 4 = x^2 - 2x + 5$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & x - 1 \\
\hline
x^2 - 2x + 5 & x^3 - 3x^2 + 7x - 5 \\
 & x^3 - 2x^2 + 5x \\
\hline
 & \quad -x^2 + 2x - 5 \\
 & \quad -x^2 + 2x - 5 \\
\hline
 & \qquad 0
\end{array}
$$

$\therefore$ $x - 1$ is a factor of $f(x)$. Therefore $1$ is another root of $f(x)$

$\boxed{1 + 2i \text{ and } 1 \text{ are the other zeroes of } f(x).}$

7. Consider the following rational function. (8 points)

$$f(x) = \frac{x^2 - 1}{2x + 1}$$

(a) Identify all the intercepts. Show your work.

(b) Find any horizontal, vertical and slant asymptotes. Show your work.

(c) Plot additional solution points as needed to sketch the graph of the rational function. Show your work.

(a) x-int

$0 = \frac{x^2-1}{2x+1}$

$\Rightarrow x^2 - 1 = 0$

$\boxed{x = \pm 1}$

y-int

$y = \frac{0^2 - 1}{2 \cdot 0 + 1} = \frac{-1}{1}$

$\boxed{y = -1}$

(b) Horizontal asymptote

$\boxed{\text{None}}$

Vertical asymptote

$2x + 1 = 0$

$\Rightarrow \boxed{x = -\frac{1}{2}}$

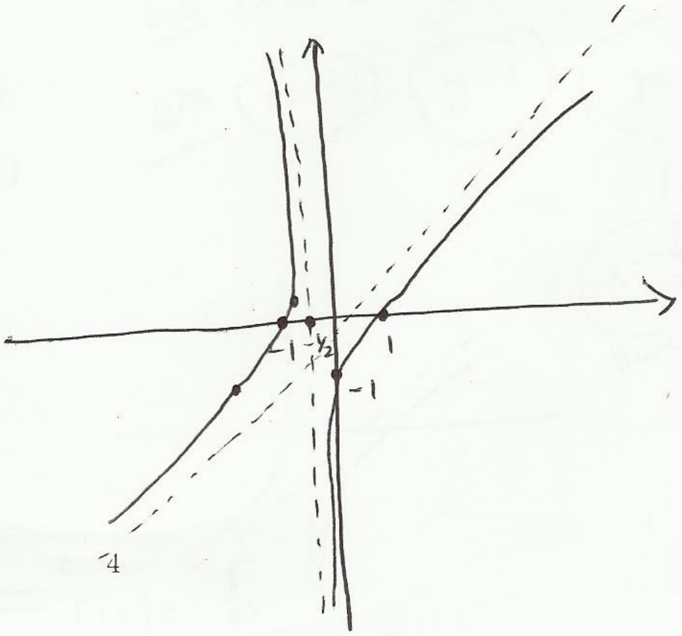
Slant asymptote

$$\begin{array}{r|l} & x/2 - 1/4 \\ \hline 2x+1 & x^2 + 0 \cdot x - 1 \\ & x^2 + \frac{x}{2} \\ \hline & -\frac{x}{2} - 1 \\ & -\frac{x}{2} - \frac{1}{4} \\ \hline & -3/4 \end{array}$$

$\boxed{y = \frac{x}{2} - \frac{1}{4}}$

(c)

| x | f(x) |
|---|---|
| -2 | -1 |
| $-\frac{3}{4}$ | $\frac{7}{8}$ |
| 0 | -1 |
| 2 | $\frac{3}{5}$ |



Bonus Question. Find two functions $f$ and $g$ such that $(f \circ g)(x) = h(x)$. Show your work. (5 points)

$$h(x) = \frac{5}{(2x-5)^3}$$

$$\boxed{\begin{aligned} f(x) &= \frac{5}{x^3} \\ g(x) &= 2x-5 \end{aligned}}$$