

1. Solve the logarithmic equation algebraically. Show your work.

(6 points)

$$\log_2(x^2 - 4x) - \log_2(x - 5) = 2$$

$$\log_2 \frac{x^2 - 4x}{x - 5} = 2$$

$$\Rightarrow \frac{x^2 - 4x}{x - 5} = 2^2 = 4$$

$$\Rightarrow x^2 - 4x = 4x - 20$$

$$\Rightarrow x^2 - 8x + 20 = 0$$

$$b^2 - 4ac = (-8)^2 - 4 \cdot 1 \cdot 20 = 64 - 80 = -16 < 0$$

∴ There are no solutions.

2. Solve the exponential equation algebraically. Show your work.

(6 points)

$$4(3^{2x-1}) - 3 = 13$$

$$4(3^{2x-1}) = 16$$

$$3^{2x-1} = 4$$

$$\ln 3^{2x-1} = \ln 4$$

$$2x - 1 (\ln 3) = \ln 4$$

$$2x - 1 = \frac{\ln 4}{\ln 3}$$

$$2x = \frac{\ln 4}{\ln 3} + 1$$

$$x = \frac{\ln 4}{2 \ln 3} + \frac{1}{2}$$

3. Find the length of the arc on a circle of radius 9 feet intercepted by a central angle of 150° . Show your work. (6 points)

$$150^\circ = \frac{5\pi}{6} = \theta, \quad r = 9 \text{ ft}$$

$$l = r\theta$$

$$l = 9 \cdot \frac{5\pi}{6} = \boxed{\frac{15\pi}{2} \text{ ft}}$$

4. (a) Find the reference angle of 1485° . Show your work.
(b) Without using a calculator evaluate the sine, cosine and tangent of the angle 1485° . Show your work. (8 points)

$$(a) \quad 1485^\circ = 360^\circ \times 4 + 45^\circ$$

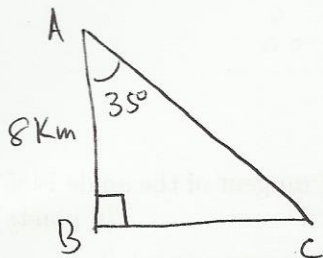
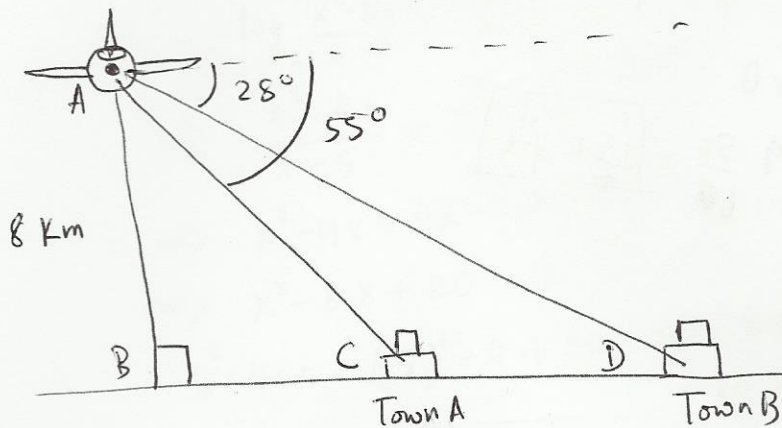
$$\therefore \text{Reference angle} = 45^\circ$$

$$(b) \quad \sin(1485^\circ) = \sin(360^\circ \times 4 + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos(1485^\circ) = \cos(360^\circ \times 4 + 45^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan(1485^\circ) = \tan(360^\circ \times 4 + 45^\circ) = \tan 45^\circ = 1$$

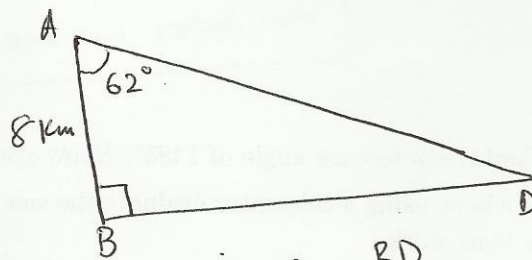
5. A passenger in an airplane at an altitude of 8 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 28° and 55° . How far apart are the towns? Show your work. (8 points)



$$\tan 35^\circ = \frac{BC}{8}$$

$$\Rightarrow BC = 8 \tan 35^\circ$$

$$= 5.6 \text{ km}$$



$$\tan 62^\circ = \frac{BD}{8}$$

$$\Rightarrow BD = 8 \tan 62^\circ$$

$$= 15 \text{ km}$$

∴ Distance between the towns

$$= 15 \text{ km} - 5.6 \text{ km} = \boxed{9.4 \text{ km}}$$

6. Verify the identity. Show your work.

(8 points)

$$\frac{\sin \theta \tan \theta}{1 - \cos \theta} - 1 = \sec \theta$$

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta \tan \theta}{1 - \cos \theta} - 1 \\ &= \frac{\sin \theta \cdot \frac{\sin \theta}{\cos \theta}}{1 - \cos \theta} - 1 \\ &= \frac{\sin^2 \theta}{\cos \theta - \cos^2 \theta} - 1 \\ &= \frac{\sin^2 \theta - \cos \theta + \cos^2 \theta}{\cos \theta (1 - \cos \theta)} \\ &= \frac{1 - \cancel{\cos \theta}}{\cos \theta (1 - \cancel{\cos \theta})} \\ &= \frac{1}{\cos \theta} = \sec \theta = \text{RHS} \end{aligned}$$

Verified

7. Find the general solution of the trigonometric equation. Show your work. (8 points)

$$2\sec^2 x + \tan^2 x - 3 = 0$$

$$2(1 + \tan^2 x) + \tan^2 x - 3 = 0$$

$$2 + 2\tan^2 x + \tan^2 x - 3 = 0$$

$$3\tan^2 x - 1 = 0$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6} + n\pi$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$x = -\frac{\pi}{6} + n\pi$$

Bonus Question. Find the exact value of the expression without using your calculator. Show your work. (5 points)

$$\frac{\tan 25^\circ + \tan 110^\circ}{1 - \tan 25^\circ \tan 110^\circ}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan(A + B)$$

$$\begin{aligned} \frac{\tan 25^\circ + \tan 110^\circ}{1 - \tan 25^\circ \tan 110^\circ} &= \tan(25^\circ + 110^\circ) = \tan(135^\circ) \\ &= -\tan(180^\circ - 135^\circ) \\ &= -\tan 45^\circ \\ &= \boxed{-1} \end{aligned}$$