Homework 1 MATH 2850

Sec 13.1

15 $r(t) = (3t+1)\hat{i} + \sqrt{3}\,t\,\hat{j} + t^2\hat{k}$

$v(t) = r'(t) = 3\hat{i} + \sqrt{3}\,\hat{j} + 2t\,\hat{k}$

$v(0) = 3\hat{i} + \sqrt{3}\,\hat{j}$ $\quad |v(0)| = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$a(t) = r''(t) = 2\hat{k}$

$a(0) = 2\hat{k}$ $\quad |a(0)| = \sqrt{2^2} = 2$

At $t=0$

$v \cdot a = |v||a|\cos\theta = 2\sqrt{3}\cdot 2\cos\theta = 4\sqrt{3}\cos\theta$

Also $v\cdot a = 3\cdot 0 + \sqrt{3}\cdot 0 + 0\cdot 2 = 0$

$\therefore\ 0 = 4\sqrt{3}\cos\theta$

$\cos\theta = 0$

$$\boxed{\theta = \frac{\pi}{2}}$$

Sec 13.2

3 $$\int_{-\pi/4}^{\pi/4} \left[(\sin t)\hat{i} + (1+\cos t)\hat{j} + (\sec^2 t)\hat{k}\right] dt$$

$$= \int_{-\pi/4}^{\pi/4} (\sin t)\,dt\,\hat{i} + \int_{-\pi/4}^{\pi/4} (1+\cos t)\,dt\,\hat{j} + \int_{-\pi/4}^{\pi/4} \sec^2 t\,dt\,\hat{k}$$

$$= -\cos t\Big|_{-\pi/4}^{\pi/4}\hat{i} + (t+\sin t)\Big|_{-\pi/4}^{\pi/4}\hat{j} + \tan t\Big|_{-\pi/4}^{\pi/4}\hat{k}$$

$$= \left(-\cos\tfrac{\pi}{4} + \cos\left(-\tfrac{\pi}{4}\right)\right)\hat{i} + \left(\tfrac{\pi}{4} + \sin\tfrac{\pi}{4} + \tfrac{\pi}{4} - \sin\left(-\tfrac{\pi}{4}\right)\right)\hat{j} + \left(\tan\tfrac{\pi}{4} - \tan\left(-\tfrac{\pi}{4}\right)\right)\hat{k}$$

$$= \boxed{\left(\frac{\pi}{2} + \sqrt{2}\right)\hat{j} + 2\hat{k}}$$

21 $v_0 = 500\ m/sec$

$\alpha = 45^\circ$

$g = 9.8\ m/sec^2$

(a) $\text{Range} = \dfrac{v_0^2}{g}\sin 2\alpha = \dfrac{500^2}{9.8}\cdot\sin 90^\circ = \boxed{25{,}510\ m}$

$\text{Flight time} = \dfrac{2v_0\sin\alpha}{g} = \dfrac{2\cdot 500\ \sin 45^\circ}{9.8} = \boxed{72\ \text{secs.}}$

(b) $x\text{-component} = (v_0\cos\alpha)t = 5000\,m.$

$500\cos 45^\circ.t = 5000$

$t = \dfrac{5000}{500\cos 45^\circ} = 14.14\ \text{secs}$

$y\text{-component} = (v_0\sin\alpha)t - \dfrac{1}{2}gt^2$

$= 500\sin 45^\circ.(14.14) - \dfrac{1}{2}.9.8.(14.14)^2$

$\approx \cancel{4950 - 960} = 4999.24 - 979.4$

$\approx \cancel{3990\ m} = \boxed{4019.84\ m}$

(c) $\text{Maximum height} = \dfrac{(v_0\sin\alpha)^2}{2g} = \dfrac{(500\sin 45^\circ)^2}{2.9.8}$

$= \dfrac{125000}{19.6} = \boxed{6377.55\ m}$

Sec 13.3

3 $r(t) = t\hat{i} + \frac{2}{3}t^{3/2}\hat{k}, \quad 0 \le t \le 8$

$r'(t) = \hat{i} + t^{1/2}\hat{k}$

Unit Tangent vector $(T) = \frac{r'(t)}{|r'(t)|} = \frac{\hat{i} + t^{1/2}\hat{k}}{\sqrt{1^2 + (t^{1/2})^2}} = \frac{\hat{i} + t^{1/2}\hat{k}}{\sqrt{1+t}}$

$$= \boxed{\frac{1}{\sqrt{1+t}}\hat{i} + \frac{\sqrt{t}}{\sqrt{1+t}}\hat{k}}$$

Arc Length $= \int_0^8 |r'(t)|\,dt$

$= \int_0^8 \sqrt{1+t}\,dt$

$= \frac{2}{3}(1+t)^{3/2}\Big|_0^8 = \frac{2}{3}\cdot 9^{3/2} - \frac{2}{3}\cdot 1^{3/2}$

$= 18 - 2/3 = \boxed{\frac{52}{3}}$

11 $s = \int_0^t |v(\tau)|\,d\tau$

$r(t) = (4\cos t)\hat{i} + (4\sin t)\hat{j} + 3t\hat{k}$

$r'(t) = v(t) = (-4\sin t)\hat{i} + (4\cos t)\hat{j} + 3\hat{k}$

$|v(t)| = \sqrt{(-4\sin t)^2 + (4\cos t)^2 + 3^2} = \sqrt{16\sin^2 t + 16\cos^2 t + 9} = \sqrt{25} = 5$

$s = \int_0^t 5\,d\tau = 5\tau\Big|_0^t = \boxed{5t}$

When $0 \le t \le \frac{\pi}{2}$ then $L = 5\cdot\frac{\pi}{2} = \boxed{\frac{5\pi}{2}}$