MATH 2850 Homework 2

Completion Points = 8
Selected problems Points = 12

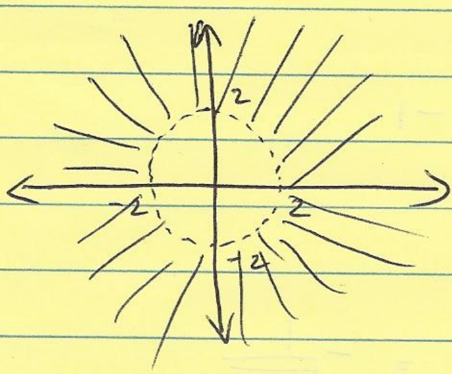
See 14.1

6 $f(x,y) = \ln(x^2+y^2-4)$

To find the domain $x^2+y^2-4 > 0$

$x^2+y^2 > 4$

Hence $D_f = \{(x,y) : x^2+y^2 > 4\}$



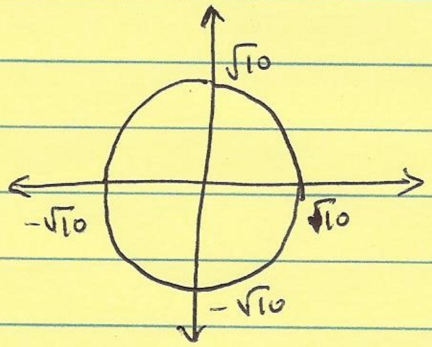
49 $f(x,y) = 16 - x^2 - y^2$ $(2\sqrt{2}, \sqrt{2})$

$f(2\sqrt{2},\sqrt{2}) = 16 - (2\sqrt{2})^2 - (\sqrt{2})^2$

$= 16 - 8 - 2 = 6$

Hence $6 = 16 - x^2 - y^2$

$x^2+y^2 = 10$ ← Level curve that passes through $(2\sqrt{2}, \sqrt{2})$



Sec 14.2

3 $\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1} = \sqrt{3^2+4^2-1} = \sqrt{25-1} = \sqrt{24} = \boxed{2\sqrt{6}}$

41 $f(x,y) = -\dfrac{x}{\sqrt{x^2+y^2}}$ $(x,y) \to (0,0)$

Keeping y constant at $y=0$

$$\lim_{x\to 0} \frac{-x}{\sqrt{x^2+0}} = \lim_{x\to 0} \frac{-x}{x} = -1$$

Now taking the path $y = mx$

$$\lim_{x\to 0} \frac{-x}{\sqrt{x^2+m^2x^2}} = \lim_{x\to 0} \frac{-x}{x\sqrt{1+m^2}} = -\frac{1}{\sqrt{1+m^2}}$$

For different values of m, we have different limits, therefore f has no limit as $(x,y) \to (0,0)$

Sec 14.3

25 $f(x,y,z) = x - \sqrt{y^2+z^2}$

$f_x = 1$

$f_y = -\frac{1}{2}(y^2+z^2)^{-1/2} \cdot 2y = -\frac{y}{\sqrt{y^2+z^2}}$

$f_z = -\frac{1}{2}(y^2+z^2)^{-1/2} \cdot 2z = \frac{-z}{\sqrt{y^2+z^2}}$

Sec 14.4

29 $z^3 - xy + yz + y^3 - 2 = 0 \qquad (1,1,1)$

$F(x,y,z) = z^3 - xy + yz + y^3 - 2$

By the Implicit Function Theorem.

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ and $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$

$F_x = -y$, $F_y = -x + z + 3y^2$, $F_z = 3z^2 + y$

$\frac{\partial z}{\partial x} = \frac{y}{3z^2+y} \qquad \frac{\partial z}{\partial y} = \frac{x-z-3y^2}{3z^2+y}$

$\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)} = \frac{1}{3+1} = \boxed{\frac{1}{4}}$, $\frac{\partial z}{\partial y} = \frac{1-1-3}{3+1} = \boxed{\frac{-3}{4}}$