MATH 2850 Homework 3 Solutions

Completion point = 10
Selected problem points = 10

Sec 14.5

15 $f(x,y,z) = xy + yz + zx$, $P_0(1,-1,2)$, $u = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\nabla f(x,y,z) = (y+z)\hat{i} + (x+z)\hat{j} + (x+y)\hat{k}$

$\nabla f(1,-1,2) = \hat{i} + 3\hat{j}$

Unit vector in the direction of $u = v = \dfrac{u}{|u|} = \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+(-2)^2}}$

$= \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$

$= \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$

$\therefore (D_v f)_{P_0} = \nabla f \cdot v$

$= (\hat{i}+3\hat{j}) \cdot \left(\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}\right)$

$= \frac{3}{7} + \frac{18}{7} = \frac{21}{7} = \boxed{3}$

Sec 14.6

7 $x+y+z=1$, $P_0(0,1,0)$

(a) Tangent Plane

$f(x,y,z) = x+y+z$

$f_x(x,y,z) = 1$, $f_y(x,y,z) = 1$, $f_z(x,y,z) = 1$

$f_x(0,1,0) = 1$, $f_y(0,1,0) = 1$, $f_z(0,1,0) = 1$

$1.(x-0) + 1.(y-1) + 1.(z-0) = 0$

$\boxed{x + y - 1 + z = 0}$

(b) Normal line

$x = 0 + 1.t \quad , \quad y = 1 + 1.t \quad , \quad z = 0 + 1.t$

$$\boxed{x = t, \; y = 1+t, \; z = t}$$

<u>20</u> $f(x,y,z) = e^x \cos yz \quad , v = 2\hat{i} + 2\hat{j} - 2\hat{k} \quad , \quad P_0(0,0,0)$

$ds = 0.1$

$\nabla f(x,y,z) = e^x \cos yz \, \hat{i} - z e^x \sin yz \, \hat{j} - y e^x \sin yz \, \hat{k}$

$\nabla f(0,0,0) = \hat{i}$

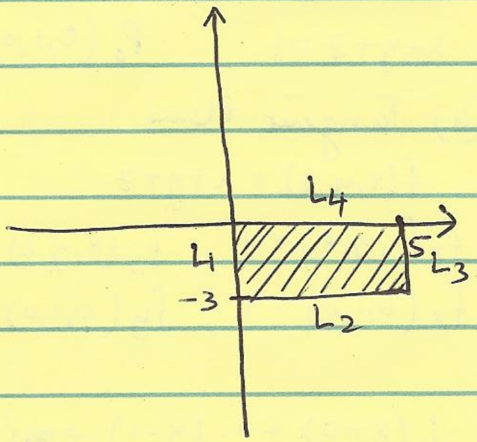
$$u = \frac{v}{|v|} = \frac{2\hat{i} + 2\hat{j} - 2\hat{k}}{\sqrt{2^2 + 2^2 + (-2)^2}} = \frac{2\hat{i} + 2\hat{j} - 2\hat{k}}{2\sqrt{3}} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} - \frac{\hat{k}}{\sqrt{3}}$$

$$\nabla f(0,0,0) \cdot u = (\hat{i}) \cdot \left(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} - \frac{\hat{k}}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$$

$$df = (\nabla f(0,0,0) \cdot u) \, ds = \frac{1}{\sqrt{3}} \cdot 0.1 = \boxed{\frac{1}{10\sqrt{3}}}$$

<u>Sec 14.7</u>

<u>35</u> $T(x,y) = x^2 + xy + y^2 - 6x + 2$



<u>Interior point</u>

$T_x = 2x + y - 6$

$T_y = x + 2y$

$2x + y - 6 = 0 \qquad x + 2y = 0$

$\qquad\qquad\qquad\qquad x = -2y$

$-4y + y = 6$

$-3y = 6$

$y = -2 \qquad x = 4$

$\boxed{(4,-2)}$ ← A candidate for points where max/min. could be attained.

$T(4,-2) = 16 - 8 + 4 - 24 + 2 = \boxed{-10}$

On L_1

$x = 0, \quad -3 \le y \le 0$

$T(0,y) = y^2 + 2$

$T'(0,y) = 2y = 0$

$y = 0$

$T(0,0) = 2$

At the endpoints

$T(0,0) = \boxed{2}, \quad T(0,-3) = \boxed{11}$

On L_2

$y = -3, \quad 0 \le x \le 5$

$T(x,-3) = x^2 - 3x + 9 - 6x + 2$

$= x^2 - 9x + 11$

$T'(x,-3) = 2x - 9 = 0$

$x = \frac{9}{2}$

$T\left(\frac{9}{2}, -3\right) = \frac{81}{4} - \frac{81}{2} + 11$

$= -\frac{81}{4} + 11 = \boxed{-\frac{37}{4}}$

At the end points

$T(0,-3) = \boxed{11}, \quad T(5,-3) = 25 - 45 + 11$

$= \boxed{-9}$

On L_3

$x = 5, \quad -3 \le y \le 0$

$T(5,y) = 25 + 5y + y^2 - 30 + 2$

$= y^2 + 5y - 3$

At the end points

$T(5,-3) = 9 - 15 - 3 = \boxed{-9}$

$T(5,0) = \boxed{-3}$

On L_4

$y = 0, \quad 0 \le x \le 5$

$T(x,0) = x^2 - 6x + 2$

$T'(x,0) = 2x - 6 = 0$

$x = 3$

$T(3,0) = 9 - 18 + 2 = \boxed{-7}$

At the endpoints

$T(0,0) = \boxed{2}, \quad T(5,0) = 25 - 30 + 2$

$= \boxed{-3}$

Comparing all the values we get

Absolute maximum = 11 attained at $(0,-3)$
Absolute minimum = -10 attained at $(4,-2)$

Sec 14.8

22 $d(x,y,z) = \sqrt{x^2+y^2+z^2}$

we minimize $f(x,y,z) = x^2+y^2+z^2$ instead of $\sqrt{x^2+y^2+z^2}$

$$g(x,y,z) = xyz - 1$$

$$\nabla f(x,y,z) = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\nabla g(x,y,z) = yz\hat{i} + xz\hat{j} + xy\hat{k}$$

$$\nabla f = \lambda \nabla g$$

$$2x\hat{i} + 2y\hat{j} + 2z\hat{k} = \lambda yz\hat{i} + \lambda xz\hat{j} + \lambda xy\hat{k}$$

$\therefore$ $2x = \lambda yz$, $2y = \lambda xz$, $2z = \lambda xy$

Multiplying the first 2 equations we get

$$4xy = \lambda^2 xyz^2$$

$\Rightarrow$ $4 = \lambda^2 z^2$ $[xy \neq 0$ since $xyz = 1]$

$\Rightarrow$ $\lambda z = \pm 2$

$$z = \frac{\pm 2}{\lambda}$$

Similarly we can also get $x = \pm \frac{2}{\lambda}$, $y = \pm \frac{2}{\lambda}$

So $xyz = 1$

$\pm \frac{8}{\lambda^3} = 1$

$\lambda^3 = \pm 8$

$\lambda = \pm 2$

$\therefore$ $x = \pm 1$, $y = \pm 1$, $z = \pm 1$

Thus the points are

$(1,1,1)$, $(1,-1,-1)$, $(-1,-1,1)$, $(-1,1,-1)$