MATH 2850

Homework 4

Completion points = 10
Selected problem points = 10

Sec 15.1

14 $\iint_R \left(\frac{\sqrt{x}}{y}\right) dA$, $R: 0 \le x \le 4, \ 1 \le y \le 2$

$$= \int_1^2 \int_0^4 \frac{\sqrt{x}}{y} \, dx \, dy$$

$$= \int_1^2 \frac{1}{y} \cdot \frac{2}{3} \cdot x^{3/2} \Big|_0^4 dy.$$

$$= \int_1^2 \frac{1}{y} \cdot \frac{2}{3} \left(4^{3/2} - 0^{3/2}\right) dy.$$

$$= \int_1^2 \frac{1}{y} \cdot \frac{16}{3} \, dy.$$

$$= \frac{16}{3} \ln y \Big|_1^2$$

$$= \frac{16}{3} (\ln 2 - \ln 1)$$

$$= \boxed{\frac{16}{3} \ln 2}$$

25 $\int_0^1 \int_0^1 2 - x - y \, dx \, dy$

$= \int_0^1 \left. 2x - \frac{x^2}{2} - yx \right|_0^1 dy$

$= \int_0^1 \left(2 - \frac{1}{2} - y\right) dy$

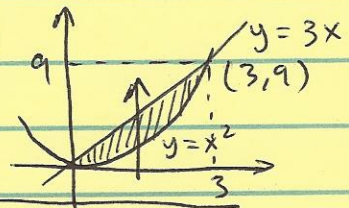
$= \int_0^1 \left(\frac{3}{2} - y\right) dy$

$= \left. \frac{3}{2}y - \frac{y^2}{2} \right|_0^1$

$= \frac{3}{2} - \frac{1}{2} = \boxed{1}$

Sec 15.2

11 (a)



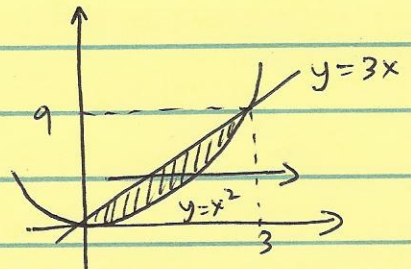
Point of intersection

$x^2 = 3x$

$x = 0, 3 \quad y = 0, 9$

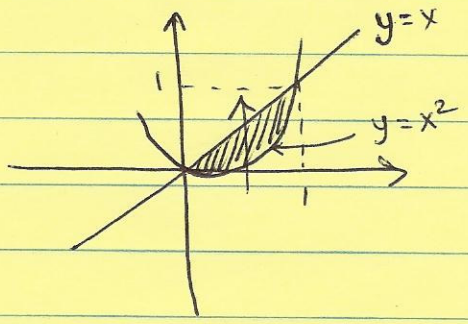
$$\int_0^3 \int_{x^2}^{3x} dy \, dx$$

(b)



$$\int_0^9 \int_{3x}^{x^2} dy \, dx$$

35 $\int_0^1 \int_y^{\sqrt{y}} dx\,dy$ $\quad y \le x \le \sqrt{y}$, $0 \le y \le 1$



Using vertical cross-sections $x^2 \le y \le x$, $0 \le x \le 1$

$$\boxed{\int_0^1 \int_{x^2}^{x} dy\,dx}$$

15.4 27 $r = 2(2-\sin 2\theta)^{1/2}$

$0 \le \theta \le \frac{\pi}{2}$

$0 \le r \le 2(2-\sin 2\theta)^{1/2}$

$$\int_0^{\pi/2} \int_0^{2(2-\sin 2\theta)^{1/2}} r\,dr\,d\theta = \int_0^{\pi/2} \frac{r^2}{2}\Big|_0^{2(2-\sin 2\theta)^{1/2}} d\theta$$

$$= \int_0^{\pi/2} \frac{4(2-\sin 2\theta)}{2}\,d\theta$$

$$= \int_0^{\pi/2} 4\,d\theta - \int_0^{\pi/2} \sin 2\theta\,d\theta$$

$$= 4\theta\Big|_0^{\pi/2} + \cos 2\theta\Big|_0^{\pi/2}$$

$$= 2\pi + (\cos\pi - \cos 0) = \boxed{2\pi - 2}$$