MATH 2850

Home Work 5

Completion points = 10
Selected problem points = 10

Sec 15.5

9 $\displaystyle\int_1^e \int_1^{e^2} \int_1^{e^3} \frac{1}{xyz}\, dx\,dy\,dz$

$= \displaystyle\int_1^e \int_1^{e^2} \frac{1}{yz} \cdot \ln x \Big|_1^{e^3} dy\,dz$

$= \displaystyle\int_1^e \int_1^{e^2} \frac{1}{yz} \left[\ln e^3 - \ln 1\right] dy\,dz$

$= \displaystyle\int_1^e \int_1^{e^2} \frac{1}{yz} \cdot 3\,dy\,dz$

$= \displaystyle 3 \int_1^e \frac{1}{z} \ln y \Big|_1^{e^2} dz$

$= \displaystyle 3 \int_1^e \frac{1}{z} \left[\ln e^2 - \ln 1\right) dz$

$= \displaystyle 3 \int_1^e 2 \frac{1}{z} \cdot dz$

$= 6 \ln z \Big|_1^e$

$= 6\left[\ln e - \ln 1\right] = \boxed{6}$

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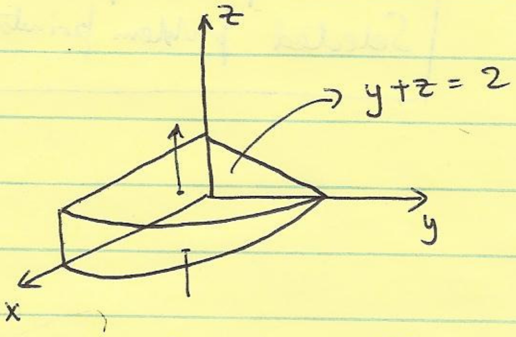
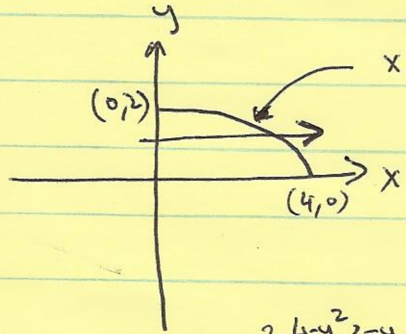


Take the projection on the xy-plane

<u>Limits for z</u> $\quad 0 \leq z \leq 2-y$



$0 \leq x \leq 4-y^2$

$0 \leq y \leq 2$

$$\therefore \quad V = \int_0^2 \int_0^{4-y^2} \int_0^{2-y} dz\,dx\,dy = \int_0^2 \int_0^{4-y^2} (2-y)\,dx\,dy = \int_0^2 2x - yx \Big|_0^{4-y^2} dy$$

$$= \int_0^2 2(4-y^2) - y(4-y^2)\,dy$$

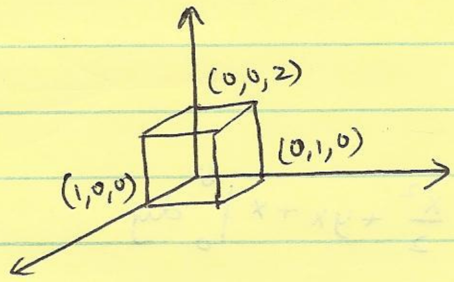
$$= \int_0^2 (8 - 2y^2 - 4y + y^3)\,dy$$

$$= 8y - \frac{2y^3}{3} - 2y^2 + \frac{y^4}{4} \Big|_0^2$$

$$= 16 - \frac{16}{3} - 8 + 4$$

$$= 12 - \frac{16}{3} = \boxed{\frac{20}{3}}$$

38 Average value $= \dfrac{1}{\text{Volume of } D} \iiint_D F\,dV$.



$0 \le x \le 1$

$0 \le y \le 1$

$0 \le z \le 2$

Volume of $D = 1 \cdot 1 \cdot 2 = 2$

$$\iiint_D F\,dV = \int_0^2\int_0^1\int_0^1 (x+y-z)\,dx\,dy\,dz$$

$$= \int_0^2\int_0^1 \frac{x^2}{2} + yx - zx \Big|_0^1 \,dy\,dz$$

$$= \int_0^2\int_0^1 \left(\frac{1}{2} + y - z\right) dy\,dz$$

$$= \int_0^2 \frac{1}{2}y + \frac{y^2}{2} - zy \Big|_0^1 dz$$

$$= \int_0^2 \left(\frac{1}{2} + \frac{1}{2} - z\right) dz$$

$$= \frac{1}{2}z + \frac{1}{2}z - \frac{z^2}{2}\Big|_0^2$$

$$= 2 - 2 = 0$$

$\therefore$ Average value of F over $D = \frac{1}{2} \cdot 0 = \boxed{0}$

Sec 15.6

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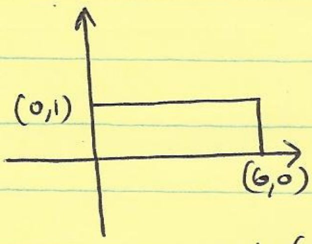

$\delta(x,y) = x+y+1$

$0 \le x \le 6$

$0 \le y \le 1$

$$M = \iint_R \delta \, dA = \int_0^1 \int_0^6 (x+y+1)\,dx\,dy = \int_0^1 \frac{x^2}{2} + yx + x \Big|_0^6 dy$$

$$= \int_0^1 (18 + 6y + 6)\,dy$$

$$= 24y + 3y^2 \Big|_0^1$$

$$= 24 + 3 = 27$$

$$M_y = \iint_R x\delta \, dA = \int_0^1 \int_0^6 x(x+y+1)\,dx\,dy = \int_0^1 \int_0^6 (x^2 + xy + x)\,dx\,dy$$

$$= \int_0^1 \frac{x^3}{3} + \frac{x^2 y}{2} + \frac{x^2}{2} \Big|_0^6 dy$$

$$= \int_0^1 (72 + 18y + 18)\,dy$$

$$= 90y + 9y^2 \Big|_0^1 = 90 + 9 = 99$$

$$M_x = \iint_R y\delta \, dA = \int_0^1 \int_0^6 y(x+y+1)\,dx\,dy = \int_0^1 \int_0^6 (yx + y^2 + y)\,dx\,dy$$

$$= \int_0^1 \frac{yx^2}{2} + y^2 x + yx \Big|_0^6 dy$$

$$= \int_0^1 (18y + 6y^2 + 6y)\,dy$$

$$= 9y^2 + 2y^3 + 3y^2 \Big|_0^1$$

$$= 9 + 2 + 3 = 14$$

$$\therefore \quad \bar{x} = \frac{M_y}{M} = \frac{99}{27} = \frac{11}{3}$$

$$\bar{y} = \frac{M_x}{M} = \frac{14}{27}$$

Center of Mass : $\boxed{\left(\frac{11}{3}, \frac{14}{27}\right)}$

$$I_y = \iint_R x^2 \delta \, dA = \int_0^1 \int_0^6 x^2 (x+y+1) \, dx \, dy$$

$$= \int_0^1 \int_0^6 (x^3 + x^2 y + x^2) \, dx \, dy$$

$$= \int_0^1 \frac{x^4}{4} + \frac{x^3 y}{3} + \frac{x^3}{3} \Big|_0^6 \, dy$$

$$= \int_0^1 (324 + 72y + 72) \, dy$$

$$= 324y + 36y^2 + 72y \Big|_0^1$$

$$= 324 + 36 + 72$$

$$= \boxed{432}$$

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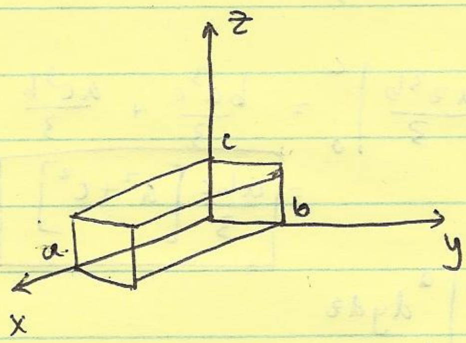

$0 \le x \le a$

$0 \le y \le b$

$0 \le z \le c$

$\delta = 1$

$$I_x = \iiint_D (y^2 + z^2) \, dV = \int_0^c \int_0^b \int_0^a (y^2 + z^2) \, dx \, dy \, dz$$

$$= \int_0^c \int_0^b y^2 x + z^2 x \Big|_0^a \, dy \, dz$$

$$= \int_0^c \int_0^b (ay^2 + az^2)\, dy\, dz$$

$$= \int_0^c \frac{ay^3}{3} + az^2 y \Big|_0^b dz$$

$$= \int_0^c \left(\frac{b^3 a}{3} + baz^2\right) dz$$

$$= \frac{b^3 a z}{3} + \frac{baz^3}{3} \Big|_0^c$$

$$= \frac{ab^3c}{3} + \frac{abc^3}{3} = \boxed{\frac{abc}{3}\left[b^2 + c^2\right]}$$

$$I_y = \int_0^c \int_0^b \int_0^a (x^2 + z^2)\, dx\, dy\, dz \quad \int_0^c \int_0^b \frac{x^3}{3} + z^2 x \Big|_0^a dy\, dz$$

$$= \int_0^c \int_0^b \left(\frac{a^3}{3} + az^2\right) dy\, dz$$

$$= \int_0^c \frac{a^3 y}{3} + az^2 y \Big|_0^b dz$$

$$= \int_0^c \left(\frac{ba^3}{3} + az^2 b\right) dz$$

$$= \frac{ba^3 z}{3} + \frac{az^3 b}{3} \Big|_0^c = \frac{ba^3 c}{3} + \frac{ac^3 b}{3}$$

$$= \boxed{\frac{abc}{3}\left[a^2 + c^2\right]}$$

$$I_z = \int_0^c \int_0^b \int_0^a (y^2 + x^2)\, dx\, dy\, dz = \int_0^c \int_0^b y^2 x + \frac{x^3}{3} \Big|_0^a dy\, dz$$

$$= \int_0^c \int_0^b \left(ay^2 + \frac{a^3}{3}\right) dy\, dz = \int_0^c \frac{ay^3}{3} + \frac{a^3 y}{3} \Big|_0^b dz$$

$$= \int_0^c \left(\frac{ab^3}{3} + \frac{a^3 b}{3}\right) dz = \frac{ab^3 c}{3} + \frac{a^3 bc}{3}$$

$$= \boxed{\frac{abc}{3}\left[b^2 + a^2\right]}$$