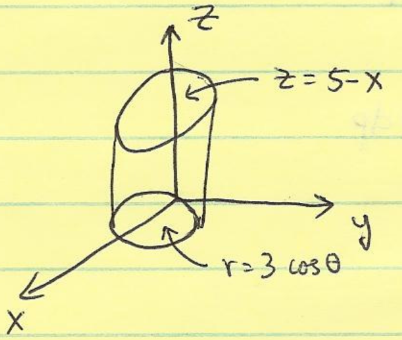
MATH 2850
Homework 6

Completion points = 10
Selected problems points = 10

Sec 15.7

16

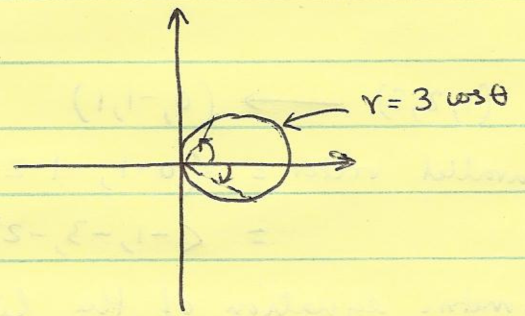


We use cylindrical coordinates here

$z = 5 - x = 5 - r\cos\theta$

$\therefore$ Limit of z: $0 \leq z \leq 5 - r\cos\theta$

Projection on the x-y plane



$0 \leq r \leq 3\cos\theta$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$$\boxed{\int_{-\pi/2}^{\pi/2} \int_0^{3\cos\theta} \int_0^{5 - r\cos\theta} dz \; r\,dr\,d\theta}$$

29 $\displaystyle\int_0^1 \int_0^{\pi} \int_0^{\pi/4} 12\rho \sin^3\phi \, d\phi\, d\theta\, d\rho$

Remember

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\therefore \sin^3\theta = \frac{3}{4}\sin\theta - \frac{\sin 3\theta}{4}$

$$= \int_0^1 \int_0^{\pi} \int_0^{\pi/4} 12\rho \left(\frac{3}{4}\sin\phi - \frac{\sin 3\phi}{4}\right) d\phi\, d\theta\, d\rho$$

$$= 3 \int_0^1 \int_0^{\pi} \int_0^{\pi/4} \rho \left(3\sin\phi - \sin 3\phi\right) d\phi\, d\theta\, d\rho$$

$$= 3\int_0^1\int_0^\pi \rho\left(-3\cos\phi + \frac{\cos 3\phi}{3}\right)\Big|_0^{\pi/4} d\theta\, d\rho$$

$$= 3\int_0^1\int_0^\pi \rho\left(-\frac{3}{\sqrt{2}} - \frac{1}{3\sqrt{2}} + 3 - \frac{1}{3}\right) d\theta\, d\rho$$

$$= 3\pi\int_0^1 \rho\left(\frac{-10}{3\sqrt{2}} + \frac{8}{3}\right) d\rho$$

$$= \frac{\cancel{3}\pi}{2}\left(\frac{-10+8\sqrt{2}}{\cancel{3}\sqrt{2}}\right)$$

$$= \boxed{\frac{\pi(8\sqrt{2}-10)}{2\sqrt{2}}} = \boxed{\pi\left(\frac{8-5\sqrt{2}}{2}\right)}$$

<u>16.1</u> <u>13</u> $(1,2,3) \longrightarrow (0,-1,1)$

Parallel vector $= \langle 0-1, -1-2, 1-3\rangle$
$= \langle -1,-3,-2\rangle$

$\therefore$ Parametric equation of the line joining $(1,2,3)$ to $(0,-1,1)$

$$C: \begin{array}{l} x = 1-t \\ y = 2-3t \\ z = 3-2t \end{array} \qquad 0 \le t \le 1$$

$v(t) = -\hat{i} - 3\hat{j} - 2\hat{k}$

$|v(t)| = \sqrt{(-1)^2+(-3)^2+(-2)^2} = \sqrt{1+9+4} = \sqrt{14}$

Line integral of f over C $= \int_0^1 F(x(t),y(t),z(t))\,|v(t)|\,dt$

$$= \int_0^1 (1-t+2-3t+3-2t)\cdot\sqrt{14}\,dt$$

$$= \int_0^1 (6-6t)\sqrt{14}\,dt = (6t-3t^2)\Big|_0^1 \sqrt{14}$$

$$= \boxed{3\sqrt{14}}$$

Sec 16.2

21 $F = z\hat{i} + x\hat{j} + y\hat{k}$

$r(t) = (\sin t)\hat{i} + (\cos t)\hat{j} + t\hat{k}, \quad 0 \le t \le 2\pi$

$$\text{Work done} = \int_C F \cdot dr = \int_0^{2\pi} \left(F(r(t)) \cdot \frac{dr}{dt} \right) dt$$

$r'(t) = (\cos t)\hat{i} - (\sin t)\hat{j} + \hat{k}$

$\cancel{|r'(t)| = \sqrt{\cos^2 t + (-\sin t)^2 + 1} = \sqrt{2}}$

$F(r(t)) = t\hat{i} + (\sin t)\hat{j} + (\cos t)\hat{k}$

$$\int_0^{2\pi} \left(F(r(t)) \cdot \frac{dr}{dt} \right) dt = \int_0^{2\pi} \left(t\hat{i} + (\sin t)\hat{j} + (\cos t)\hat{k} \right) \cdot \left(\cos t\,\hat{i} - \sin t\,\hat{j} + \hat{k} \right) dt$$

$$= \int_0^{2\pi} (t\cos t - \sin^2 t + \cos t)\, dt$$

$$= \int_0^{2\pi} t\cos t\, dt - \int_0^{2\pi} \sin^2 t\, dt + \int_0^{2\pi} \cos t\, dt$$

$$= t\sin t + \cos t \Big|_0^{2\pi} - \int_0^{2\pi} \frac{1 - \cos 2t}{2}\, dt + \sin t \Big|_0^{2\pi}$$

$$= (2\pi \sin 2\pi + \cos 2\pi - \cos 0) - \left(\frac{1}{2}t - \frac{\sin 2t}{4} \right)\Big|_0^{2\pi} + 0$$

$$= (1 - 1) - \left(\frac{1}{2} \cdot 2\pi - \frac{\sin 4\pi}{4} \right) + 0$$

$$= \boxed{-\pi}$$

49 $F = (x-z)\hat{i} + x\hat{k}$

$r(t) = (\cos t)\hat{i} + (\sin t)\hat{k}$, $0 \le t \le \pi$

$$\text{Flow} = \int_C F \cdot T\, ds = \int_0^{\pi} \left(F(r(t)) \cdot \frac{dr}{dt} \right) dt$$

$r'(t) = (-\sin t)\hat{i} + (\cos t)\hat{k}$

$F(r(t)) = (\cos t - \sin t)\hat{i} + (\cos t)\hat{k}$

$$\int_0^{\pi} \left(F(r(t)) \cdot \frac{dr}{dt} \right) dt = \int_0^{\pi} \left((\cos t - \sin t)\hat{i} + (\cos t)\hat{k} \right) \cdot \left((-\sin t)\hat{i} + (\cos t)\hat{k} \right) dt$$

$$= \int_0^{\pi} \left(-\cos t \sin t + \sin^2 t + \cos^2 t \right) dt$$

$$= \int_0^{\pi} (1 - \cos t \sin t)\, dt$$

$$= t \Big|_0^{\pi} - \frac{1}{2} \int_0^{\pi} 2 \cos t \sin t\, dt$$

$$= \pi - \frac{1}{2} \int_0^{\pi} \sin 2t\, dt$$

$$= \pi + \frac{1}{2} \cdot \frac{\cos 2t}{2} \Big|_0^{\pi}$$

$$= \boxed{\pi}$$