MATH 2850 Sec 007

ELEMENTARY MULTIVARIABLE CALCULUS

QUIZ 2

September 20, 2012

Name (Last, First) Key.

1. Find $\partial w/\partial v$ when $u = -1, v = 2$ if $w = xy + \ln z, x = v^2/u, y = u + v, z = \cos u$.

By chain rule $\dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\cdot\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\cdot\dfrac{\partial y}{\partial v} + \dfrac{\partial w}{\partial z}\cdot\dfrac{\partial z}{\partial v}$

$= y\cdot\dfrac{2v}{u} + x\cdot 1 + \dfrac{1}{z}\cdot 0$

$= \dfrac{2vy}{u} + x$

When $u=-1, v=2$

$x = \dfrac{v^2}{u} = \dfrac{4}{-1} = -4$

$y = u+v = -1+2 = 1$

$= \dfrac{2\cdot 2\cdot 1}{-1} + (-4) = -4-4 = \boxed{-8}$

2. Find the derivative of the function at P_0 in the direction of $\mathbf{u}$.

$f(x,y,z) = x^2 + 2y^2 - 3z^2,\quad P_0(1,1,1),\quad \mathbf{u} = \mathbf{i}+\mathbf{j}+\mathbf{k}$

$(D_{\mathbf{v}}f)_{(1,1,1)} = (\nabla f)_{(1,1,1)}\cdot\mathbf{v}$ where $\mathbf{v} = \dfrac{\mathbf{u}}{|\mathbf{u}|} = \dfrac{\hat{i}}{\sqrt{3}} + \dfrac{\hat{j}}{\sqrt{3}} + \dfrac{\hat{k}}{\sqrt{3}}$

$\nabla f = 2x\hat{i} + 4y\hat{j} - 6z\hat{k}$

$\nabla f(1,1,1) = 2\hat{i} + 4\hat{j} - 6\hat{k}$

$(D_{\mathbf{v}}f)_{(1,1,1)} = (2\hat{i} + 4\hat{j} - 6\hat{k})\cdot\left(\dfrac{\hat{i}}{\sqrt{3}} + \dfrac{\hat{j}}{\sqrt{3}} + \dfrac{\hat{k}}{\sqrt{3}}\right)$

$= \dfrac{2}{\sqrt{3}} + \dfrac{4}{\sqrt{3}} - \dfrac{6}{\sqrt{3}} = \boxed{0}$