

MATH 2850 Sec 007  
ELEMENTARY MULTIVARIABLE CALCULUS

QUIZ 5

November 8, 2012

Name (Last, First) \_\_\_\_\_

Key

1. Find the line integral along the given path  $C$ .

$$\int_C \frac{x}{y^2} dx, \quad C: x=t^2, y=t, \quad 1 \leq t \leq 2.$$

$$= \int_1^2 \frac{t^2}{t^2} 2t dt$$

$$= \int_1^2 2t dt = t^2 \Big|_1^2 = 4 - 1 = \boxed{3}$$

2. Use Green's Theorem to find the outward flux for the field  $\mathbf{F}$  and the curve  $C$ .

$$\mathbf{F} = (x^2 + 4y)\mathbf{i} + (x + y^2)\mathbf{j}$$

$C$ : The square bounded by  $x=0, x=1, y=0, y=1$ . ( $\text{div } \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$ )

$$\text{Outward Flux} = \int_C \mathbf{F} \cdot \mathbf{n} ds = \iint_R \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dA$$

$$= \int_0^1 \int_0^1 (2x + 2y) dx dy$$

$$= \int_0^1 (x^2 + 2yx) \Big|_0^1 dy$$

$$= \int_0^1 (1 + 2y) dy = y + y^2 \Big|_0^1 = \boxed{2}$$

$M = x^2 + 4y$   
 $\frac{\partial M}{\partial x} = 2x$   
 $N = x + y^2$   
 $\frac{\partial N}{\partial y} = 2y$