MATH 2850 Sec 007

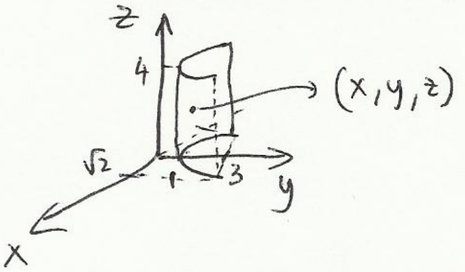
ELEMENTARY MULTIVARIABLE CALCULUS

QUIZ 6

November 9, 2012

Name (Last, First) Key

1. Find a parametrization of the surface cut from the **parabolic cylinder** $y = x^2 + 1$ by the planes $z = 0$, $z = 4$ and $y = 3$. Show your work.



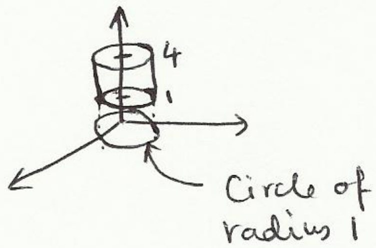
$y = 3 \Rightarrow 3 = x^2 + 1$

$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

$$\boxed{\begin{array}{ll} x = t & -\sqrt{2} \le t \le \sqrt{2} \\ y = t^2 + 1 & 0 \le z \le 4 \\ z = z & \end{array}}$$

2. Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral. Additionally verify your answer with the usual formula for the surface area of a sides of a cylinder ($S = 2\pi rh$).

   The portion of the cylinder $x^2 + y^2 = 1$ between the planes $z = 1$ and $z = 4$.



$x = \cos\theta \qquad 0 \le \theta \le 2\pi$

$y = \sin\theta \qquad 1 \le z \le 4$

$z = z$

$\therefore r(\theta, z) = (\cos\theta)\hat{i} + (\sin\theta)\hat{j} + (z)\hat{k}$

$r_\theta = (-\sin\theta)\hat{i} + (\cos\theta)\hat{j} + \hat{k}$

$r_z = \hat{k}$

$$r_\theta \times r_z = \begin{vmatrix} i & j & k \\ -\sin\theta & \cos\theta & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$= \hat{i}(\cos\theta) - \hat{j}(-\sin\theta) + \hat{k}(0-0)$

$= \hat{i}(\cos\theta) + \hat{j}(\sin\theta)$

$|r_\theta \times r_z| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

Surface area $= \int_1^4 \int_0^{2\pi} 1 \, d\theta \, dz$

$= 2\pi \int_1^4 dz = \boxed{6\pi}$

Using formula $2\pi rh$

$h = 3$, $r = 1$

$\therefore 2\pi rh = \boxed{6\pi}$