MATH 2850 Sec 007
ELEMENTARY MULTIVARIABLE CALCULUS
QUIZ 8

December 7, 2012

Name (Last, First) Key.

1. Consider the field $\mathbf{F} = (x-y)\mathbf{i} + (y-z)\mathbf{j} + (z-x)\mathbf{k}$. Find the Curl $\nabla \times \mathbf{F}$ of this field. Show your work.

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-y & y-z & z-x \end{vmatrix} = i\left(0-(-1)\right) + j\left(0-(-1)\right) + k\left(0-(-1)\right) = \boxed{i+j+k}$$

2. Using the above result evaluate

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, d\sigma$$

where S is the surface with the parametric equations

$S:\ \mathbf{r}(r,\theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (5-r)\mathbf{k},\ \ 0 \le r \le 5,\ \ 0 \le \theta \le 2\pi$

$$r_r = \langle \cos\theta, \sin\theta, -1 \rangle$$

$$r_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$$

$$r_r \times r_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & -1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = i\left(0 + r\cos\theta\right) + j\left(r\sin\theta - 0\right) + k\left(r\cos^2\theta + r\sin^2\theta\right) = \langle r\cos\theta, r\sin\theta, r \rangle$$

$$\iint_S (\nabla \times F)\cdot n\, d\sigma = \int_0^{2\pi}\int_0^5 \langle 1,1,1\rangle \cdot \langle r\cos\theta, r\sin\theta, r\rangle\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^5 \left(r\cos\theta + r\sin\theta + r\right) dr\, d\theta$$

back

$$= \int_0^{2\pi} \frac{r^2}{2}\cos\theta + \frac{r^2}{2}\sin\theta + \frac{r^2}{2} \Big|_0^5 \, d\theta$$

$$= \frac{25}{2} \int_0^{2\pi} (\cos\theta + \sin\theta + 1) \, d\theta$$

$$= \frac{25}{2} \left[\sin\theta - \cos\theta + \theta \Big|_0^{2\pi} \right]$$

$$= \frac{25}{2} \left[0 - 0 - 1 + 1 + 2\pi \right]$$

$$= \boxed{25\pi}$$