

$$17 \quad f(x,y) = 4y^4 - 32y^2 + 8x^2 - 16$$

$$f_x(x,y) = 16x$$

$$f_y(x,y) = 16y^3 - 64y$$

$$16x = 0 \Rightarrow x = 0$$

$$16y^3 - 64y = 0 \Rightarrow 16y(y^2 - 4) = 0$$
$$\Rightarrow y = 0, 2, -2$$

o Critical points:  $(0,0), (0,2), (0,-2)$

$$f_{xx}(x,y) = 16$$

$$f_{yy}(x,y) = 48y^2 - 64$$

$$f_{xy}(x,y) = 0$$

$$D(0,0) = f_{xx}(0,0) \cdot f_{yy}(0,0) - [f_{xy}(0,0)]^2$$
$$= 16 \cdot (-64) - 0^2 = -1024 < 0$$

o  $f(0,0)$  is a saddle point.

$$D(0,2) = f_{xx}(0,2) \cdot f_{yy}(0,2) - [f_{xy}(0,2)]^2$$
$$= 16 \cdot 128 - 0^2$$
$$= 2048 > 0$$

$$f_{xx}(0,2) = 16 > 0$$

o  $(0,2)$  is a point of local minimum

$$\text{Local min} = f(0,2)$$
$$= 4 \cdot 2^4 - 32 \cdot 2^2 - 8 \cdot 0^2 - 16$$
$$= 64 - 128 - 16$$
$$= \boxed{-80}$$

$$D(0,-2) = f_{xx}(0,-2) \cdot f_{yy}(0,-2) - [f_{xy}(0,-2)]^2$$
$$= 16 \cdot 128 - 0^2 = 2048 > 0$$

$$f_{xx}(0,-2) = 16 > 0$$

o  $(0,-2)$  is a point of local minimum

$$\text{Local min} = f(0,-2)$$
$$= 4 \cdot (-2)^4 - 32(-2)^2 + 8 \cdot 0^2 - 16$$
$$= 64 - 128 - 16$$
$$= \boxed{-80}$$