

22 Augmented matrix $[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array} \right]$

$R_2 - 2R_1 \rightarrow R_2$

$-2R_1 = [-2 \ -4 \ -2 \ | \ -2 \ 0 \ 0]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -4 & 3 & -2 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array} \right]$$

$R_1 - 2R_3 \rightarrow R_1$

$-2R_3 = [0 \ -2 \ 6 \ | \ 0 \ 0 \ -2]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & -2 \\ 0 & -4 & 3 & -2 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array} \right]$$

$4R_3 + R_2 \rightarrow R_2$

$4R_3 = [0 \ 4 \ -12 \ | \ 0 \ 0 \ 4]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & -2 \\ 0 & -4 & 3 & -2 & 1 & 0 \\ 0 & 0 & -9 & -2 & 1 & 4 \end{array} \right]$$

$3R_2 + R_3 \rightarrow R_2$

$3R_2 = [0 \ -12 \ 9 \ | \ -6 \ 3 \ 0]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & -2 \\ 0 & -12 & 0 & -8 & 4 & 4 \\ 0 & 0 & -9 & -2 & 1 & 4 \end{array} \right]$$

$9R_1 + 5R_3 \rightarrow R_1$

$9R_1 = [9 \ 0 \ 45 \ | \ 9 \ 0 \ -18]$
 $5R_3 = [0 \ 0 \ -45 \ | \ -10 \ 5 \ 20]$

$$\left[\begin{array}{ccc|ccc} 9 & 0 & 0 & -1 & 5 & 2 \\ 0 & -12 & 0 & -8 & 4 & 4 \\ 0 & 0 & -9 & -2 & 1 & 4 \end{array} \right]$$

$\frac{1}{9}R_1 \rightarrow R_1, \frac{1}{12}R_2 \rightarrow R_2, -\frac{1}{9}R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{9} & \frac{5}{9} & \frac{2}{9} \\ 0 & 1 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{9} & -\frac{1}{9} & -\frac{4}{9} \end{array} \right]$$

$A^{-1} = \begin{bmatrix} -\frac{1}{9} & \frac{5}{9} & \frac{2}{9} \\ \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{9} & -\frac{1}{9} & -\frac{4}{9} \end{bmatrix}$

$B = \begin{bmatrix} 2 \\ 5 \\ -7 \end{bmatrix}$

$X = A^{-1}B = \begin{bmatrix} -\frac{1}{9} & \frac{5}{9} & \frac{2}{9} \\ \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{9} & -\frac{1}{9} & -\frac{4}{9} \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ -7 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$