14 $\displaystyle\int_0^1 t^3(1+t^4)^3\,dt$

Let $u = 1+t^4$

$\dfrac{du}{dt} = 4t^3 \Rightarrow \dfrac{1}{4}du = t^3\,dt$

When $t=0$, $u=1$

$t=1$, $u=2$

$$= \int_1^2 u^3 \cdot \frac{1}{4}\,du$$

$$= \frac{1}{4}\int_1^2 u^3\,du$$

$$= \left.\frac{u^4}{16}\right|_1^2$$

$$= \frac{2^4}{16} - \frac{1^4}{16}$$

$$= 1 - \frac{1}{16} = \boxed{\frac{15}{16}}$$

15 $\displaystyle\int_{\sqrt{2}}^1 \left(\frac{u^7}{6} - \frac{1}{u^4}\right) du$

$$= \int_{\sqrt{2}}^1 \frac{u^7}{6}\,du - \int_{\sqrt{2}}^1 \frac{1}{u^4}\,du$$

$$= \frac{1}{6}\int_{\sqrt{2}}^1 u^7\,du - \int_{\sqrt{2}}^1 \frac{1}{u^4}\,du$$

$$= \left.\frac{u^8}{48}\right|_{\sqrt{2}}^1 - \left.\frac{u^{-3}}{-3}\right|_{\sqrt{2}}^1$$

$$= \left(\frac{1^8}{48} - \frac{(\sqrt{2})^8}{48}\right) + \left(\frac{1}{3.(1)^3} - \frac{1}{3.(\sqrt{2})^3}\right)$$

$$= \frac{1}{48} - \frac{16}{48} + \left(\frac{1}{3} - \frac{1}{6\sqrt{2}}\right) = -\frac{15}{48} + \frac{1}{3} - \frac{1}{6\sqrt{2}}$$

$$= \frac{1}{48} - \frac{1}{6\sqrt{2}} = \boxed{\frac{\sqrt{2}-8}{48\sqrt{2}}}$$