2

1. Solve the following equations for all x such that $0 \leq x < 2\pi$. (7 points)

$$\cos x + \cos 2x = 0$$

Show your work.

$\cos 2x = 2\cos^2 x - 1$

$\cos x + 2\cos^2 x - 1 = 0$

$2\cos^2 x + \cos x - 1 = 0$

Quadratic equation in $\cos x$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

x is in Ist, IVth Quad

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

$\cos x + 1 = 0$

$\cos x = -1$

$\boxed{x = \pi}$

2. Evaluate the following limit. Show your work. (5 points)

$$\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$$

$$\lim_{x\to 0} \frac{(\sqrt{x^2+9}-3)}{x^2} \frac{(\sqrt{x^2+9}+3)}{(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{x^2+\cancel{9}-\cancel{9}}{x^2(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{x^2+9}+3)} = \lim_{x\to 0} \frac{1}{\sqrt{x^2+9}+3}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

3. Let

$$f(x) = \begin{cases} x^2 + 2, & \text{for } x \le 0 \\ x + c, & \text{for } x > 0 \end{cases}$$

(a) Graph $f(x)$ when $c = 1$, and determine whether $f(x)$ is continuous for this choice of c.

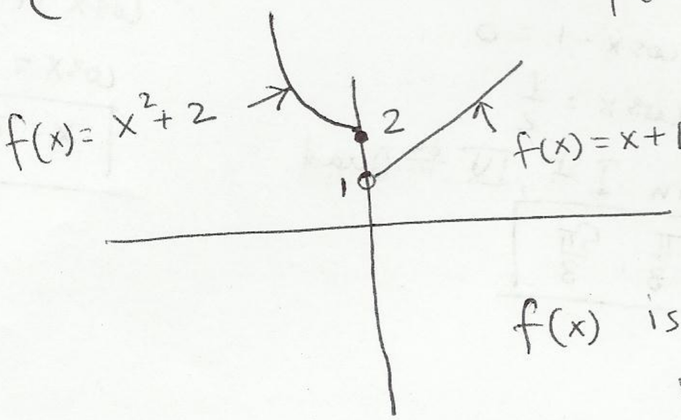
(b) How must you choose c so that $f(x)$ is continuous for all $x \in (-\infty, \infty)$?. Show your work.

(10 points)

(a) $f(x) = \begin{cases} x^2+2, & \text{for } x \le 0 \\ x+c, & \text{for } x > 0 \end{cases}$

When $c = 1$ then

$f(x) = \begin{cases} x^2+2, & \text{for } x \le 0 \\ x+1, & \text{for } x > 0 \end{cases}$



$f(x)$ is not continuous at $x = 0$.

(b) For $f(x)$ to be continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

For $\lim_{x \to 0} f(x)$ to exist $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$$\lim_{x \to 0^+} (x + c) = \lim_{x \to 0^-} x^2 + 2$$

$$\boxed{c = 2}$$

4. Evaluate the following limits. Show your work. (8 points)

(a) $\lim_{x\to-\infty} \frac{5x^2-3x+1}{3-2x^2}$ (b) $\lim_{x\to 0} \frac{1-\cos(5x)}{2x}$

(a) $\lim_{x\to-\infty} \frac{5-\frac{3}{x}+\frac{1}{x^2}}{\frac{3}{x^2}-2}$ [Dividing the numerator and denominator by x^2]

$= \frac{5-0+0}{0-2}$

$= \boxed{-\frac{5}{2}}$

$\left(\lim_{x\to 0} \frac{1-\cos 5x}{5x}\right)\cdot \frac{5x}{2x}$

$= 0 \cdot \frac{5}{2} = \boxed{0}$

5. (a) Use the **formal definition** to find the derivative of $y=-2x^2$ at $x=1$.

(b) Find the equation of the tangent line at the point (1, -2). Show your work. (10 points)

(a) $f(x) = -2x^2$

$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{-2(1+h)^2-(-2)}{h}$

$= \lim_{h\to 0} \frac{-2(1+2h+h^2)+2}{h}$

$= \lim_{h\to 0} \frac{\cancel{-2}-4h-2h^2+\cancel{2}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(-4-2h)}{\cancel{h}}$

$= \lim_{h\to 0} -4-2h = \boxed{-4}$

(b) $f(1) = -2 = y$

Point slope form : $y-(-2) = -4(x-1)$

$y+2 = -4x+4$

$\boxed{y=-4x+2}$

6. Differentiate with respect to x. Show your work. (5 points)

$$f(x) = \frac{x^3}{15} - \frac{x^4}{20} + \frac{2}{15}$$

$$f'(x) = \frac{3x^2}{15} - \frac{4x^3}{20} + 0$$

$$= \boxed{\frac{x^2}{5} - \frac{x^3}{5}}$$

7. Find the coordinate(s) of all the points on the graph of $y = f(x)$ that have a horizontal tangent. Show your work. (Hint : Think about the slope of a horizontal tangent). (5 points)

$$f(x) = 5x - x^2$$

Horizontal tangents have slope 0.

$$\therefore f'(x) = 5 - 2x$$

$$f'(x) = 0$$

$$5 - 2x = 0$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$y = 5 \cdot \frac{5}{2} - \left(\frac{5}{2}\right)^2$$

$$= \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

The point is $\left(\frac{5}{2}, \frac{25}{4}\right)$