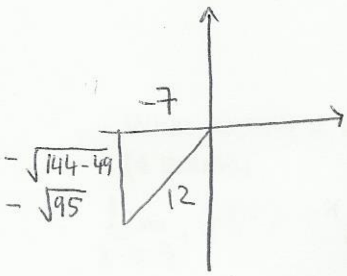
1. Find the other two trigonometric functions ($\sin x, \tan x$), if x lies in the specified interval. Show your work. (4 points)



$$\cos x = -\frac{7}{12}, \quad x \in \left[\pi, \frac{3\pi}{2}\right]$$

$\sin x = \boxed{\frac{-\sqrt{95}}{12}}$

$\tan x = \frac{-\sqrt{95}}{-7} = \boxed{\frac{\sqrt{95}}{7}}$

2. Find the domain and range of the given function. Show your work. (4 points)

$$g(x) = \sqrt{36 - x^2}$$

$36 - x^2 \geq 0$

$x^2 \leq 36$

$-6 \leq x \leq 6$

Domain$(g) = [-6, 6]$

Range $(g) = [0, 6]$

$0 \leq \sqrt{36 - x^2} \leq 6$

3. Solve for y in terms of x. Show your work. (4 points)

$$\ln(2y-1) - \ln 5 = x + \ln x$$

$$\ln\left(\frac{2y-1}{5}\right) = x + \ln x$$

$$e^{\ln\left(\frac{2y-1}{5}\right)} = e^{x+\ln x}$$

$$\frac{2y-1}{5} = e^x \cdot e^{\ln x} = xe^x$$

$$2y-1 = 5xe^x$$

$$2y = 5xe^x + 1$$

$$\boxed{y = \frac{5xe^x+1}{2}}$$

4. Evaluate the following limits. Show your work. (4 points)

(a) $\lim\limits_{h\to 0} \dfrac{h}{\sqrt{4h+9}-3}$, (b) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{\sin 4x}$

$$= \lim_{h\to 0} \frac{h(\sqrt{4h+9}+3)}{(\sqrt{4h+9}-3)(\sqrt{4h+9}+3)}$$

$$= \lim_{h\to 0} \frac{h(\sqrt{4h+9}+3)}{4h+\cancel{9}-\cancel{9}}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(\sqrt{4h+9}+3)}{4\cancel{h}}$$

$$= \frac{\sqrt{4\cdot 0+9}+3}{4} = \frac{\sqrt{9}+3}{4}$$

$$= \frac{3+3}{4} = \frac{6}{4} = \boxed{\frac{3}{2}}$$

$$= \lim_{x\to 0} \frac{\frac{\sin 3x}{3x}\cdot 3x}{\frac{\sin 4x}{4x}\cdot 4x}$$

$$= \frac{\lim\limits_{3x\to 0} \frac{\sin 3x}{3x}}{\lim\limits_{4x\to 0} \frac{\sin 4x}{4x}} \cdot \lim_{x\to 0} \frac{3x}{4x}$$

$$= \frac{1}{1} \cdot \frac{3}{4} = \boxed{\frac{3}{4}}$$

5. Let

$$f(x) = \begin{cases} 6 - x, & x < 3 \\ \dfrac{x}{2} + a, & x \geq 3 \end{cases}$$

What should a be, in order for $f(x)$ to be continuous at $x = 3$. Show your work. (4 points)

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x}{2} + a = \frac{3}{2} + a$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} 6-x = 6-3=3$

$f(3) = \frac{3}{2} + a$

For $\lim_{x\to 3} f(x)$ to exist $\quad \frac{3}{2} + a = 3$

$\Rightarrow \quad a = 3 - \frac{3}{2} = \frac{3}{2}$

$f(3) = \frac{3}{2} + \frac{3}{2} = 3 = \lim_{x\to 3} f(x)$

$\therefore \boxed{a = \frac{3}{2}}$

6. Let $f(x) = 3x^3 - 2$. Find $f^{-1}(x)$ and identify the domain and range of $f^{-1}(x)$. To check the answer, determine whether $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. Show your work. (5 points)

$f(x) = 3x^3 - 2$

$y = 3x^3 - 2$

$x = 3y^3 - 2$

$x + 2 = 3y^3$

$y^3 = \frac{x+2}{3}$

$y = \sqrt[3]{\frac{x+2}{3}}$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+2}{3}}}$

$\boxed{\text{Domain}(f^{-1}) = (-\infty, \infty)} = \text{Range of } (f)$

$\text{Domain}(f) = \boxed{(-\infty, \infty) = \text{Range of } (f^{-1})}$

$f(f^{-1}(x)) = f\left(\sqrt[3]{\frac{x+2}{3}}\right) = 3\left(\sqrt[3]{\frac{x+2}{3}}\right)^3 - 2$

$= \cancel{3}\left(\frac{x+2}{\cancel{3}}\right) - 2$

$= x$

$f^{-1}(f(x)) = f^{-1}(3x^3 - 2) = \sqrt[3]{\frac{3x^3 \cancel{-2} \cancel{+2}}{3}}$

$= \sqrt[3]{\frac{\cancel{3}x^3}{\cancel{3}}} = \sqrt[3]{x^3}$

$= x$

7. Find the horizontal, vertical and oblique asymptotes of the graph of the following function. Show your work. (5 points)

$$y = \frac{2x^2 + 3x - 1}{x + 1}$$

$\lim_{x \to \infty} \frac{2x^2+3x-1}{x+1} = \infty$ Therefore no horizontal asymptotes

$\lim_{x \to -\infty} \frac{2x^2+3x-1}{x+1} = -\infty$

$x+1 = 0$

$\boxed{x = -1}$ is the vertical asymptote

$$\begin{array}{r|l} & 2x + 1 \\ \hline x+1 & 2x^2+3x-1 \\ & 2x^2+2x \\ \hline & \quad x - 1 \\ & \quad x + 1 \\ \hline & \quad\; -2 \end{array}$$

Therefore $\boxed{y = 2x+1}$ is the oblique asymptote

8. Evaluate the limit. Show your work. (4 points)

$$\lim_{x\to\infty} \sqrt[4]{\frac{7+81x^2}{x^2+5}}$$

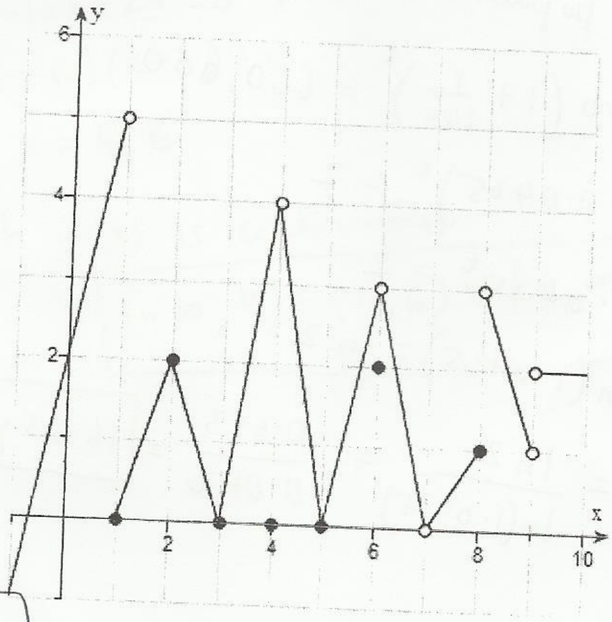
$= \lim_{x \to \infty} \sqrt[4]{\frac{\frac{7}{x^2}+81}{1+\frac{5}{x^2}}}$

$= \sqrt[4]{\frac{0+81}{1+0}} = \sqrt[4]{81} = \boxed{3}$

9. For the graph of $f(x)$ graphed below, find the following limits, if they exist. Show your work.

(4 points)

(a) $\lim_{x \to 4} f(x)$ (b) $\lim_{x \to 1} f(x)$ (c) $\lim_{x \to 7} f(x)$



(a) $\boxed{\lim_{x \to 4} f(x) = 4}$

(b) $\lim_{x \to 1^-} f(x) = 5$

$\lim_{x \to 1^+} f(x) = 0$

Since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

Therefore $\lim_{x \to 1} f(x)$ does not exist.

(c) $\boxed{\lim_{x \to 7} f(x) = 0}$

10. The population of a city is 330,000 and is increasing at a rate of 4.25% each year. Approximately when will the population reach 660,000? Show your work. (4 points)

Initial population = 330,000 $\quad r = 4.25\%$

Final population = 660,000

$$330{,}000\left(1+\frac{r}{100}\right)^t = 660{,}000$$

$$(1+0.0425)^t = 2$$

$$1.0425^t = 2$$

$$t \ln(1.0425) = \ln 2$$

$$t = \frac{\ln 2}{\ln(1.0425)} = \frac{0.693}{0.0416} = \boxed{16.65 \text{ yrs}}$$

11. Simplify using the properties of logarithms and write the expression as a single term. Show your work. (4 points)

$$\ln(\frac{x^2\sqrt{y}}{z}) - \frac{1}{2}\ln(x) + \ln(x^3y)$$

$$= \ln\left(\frac{x^2\sqrt{y}}{z}\right) - \ln\sqrt{x} + \ln(x^3y)$$

$$= \ln\left(\frac{x^2\sqrt{y}}{z\sqrt{x}} \cdot x^3y\right)$$

$$= \ln\left(\frac{x^5 y^{3/2}}{z\sqrt{x}}\right) = \boxed{\ln\left(\frac{x^{9/2}y^{3/2}}{z}\right)}$$

12. Determine the points at which the following function is continuous. Show your work. (4 points)

$$f(x) = \frac{x+6}{x^2 - 10x + 24} = \frac{x+6}{(x-4)(x-6)}$$

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$

$x = 4, 6$

Points at which $f(x)$ is continuous

$= \boxed{(-\infty, 4) \cup (4, 6) \cup (6, \infty)}$