1. Find the derivative of y with respect to x. Show your work. (6 points)
(Hint: $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$)

$$y = \sin^{-1}(3x^2 - 1)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(3x^2-1)^2}} \cdot 6x = \frac{6x}{\sqrt{6x^2-9x^4}}$$

$$= \frac{6\cancel{x}}{\cancel{x}\sqrt{6-9x^2}}$$

$$= \boxed{\frac{6}{\sqrt{6-9x^2}}}$$

2. Assume that all variables are implicit functions of time t. Find $\frac{dy}{dt}$. Show your work. (6 points)

$$2x^2 - 3y^2 + 4xy = -43; \quad \frac{dx}{dt} = 6 \text{ when } x = 2 \text{ and } y = -3.$$

$$4x\frac{dx}{dt} - 6y\frac{dy}{dt} + 4x\frac{dy}{dt} + 4y\frac{dx}{dt} = 0$$

$$\frac{dy}{dt}(4x-6y) = -4x\frac{dx}{dt} - 4y\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{(-4x-4y)\frac{dx}{dt}}{4x-6y} = \frac{(-8+12)\cdot 6}{8+18}$$

$$= \frac{4\cdot 6}{26} = \frac{24}{26} = \boxed{\frac{12}{13}}$$

3. A spherical balloon is inflating with helium at a rate of 64π ft^3/min. How fast is the balloon's radius increasing at the instant the radius is 2 ft? How fast is the surface area increasing? ($V = \frac{4}{3}\pi r^3, S = 4\pi r^2$). Show your work. (8 points)

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\cancel{4}\cancel{64}\cancel{\pi} = \cancel{4}\cancel{\pi}\cancel{4} \cdot \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = 4 \text{ ft/min}}$$

$$S = 4\pi r^2$$

$$\frac{ds}{dt} = 4\pi \cdot 2r\frac{dr}{dt}$$

$$= 4 \cdot \pi \cdot 2 \cdot 2 \cdot 4 \cdot$$

$$\boxed{\frac{ds}{dt} = 64\pi \text{ ft}^2/\text{min}}$$

4. Find the linearization $L(x)$ of $f(x) = \tan x$ at $x = \frac{3\pi}{4}$. Show your work. (8 points)

$$f'(x) = \sec^2 x \quad , \quad f'\left(\frac{3\pi}{4}\right) = \sec^2 \frac{3\pi}{4} = 2$$

$$L(x) = f\left(\frac{3\pi}{4}\right) + f'\left(\frac{3\pi}{4}\right)\left(x - \frac{3\pi}{4}\right)$$

$$f\left(\frac{3\pi}{4}\right) = \tan \frac{3\pi}{4} = -1$$

$$L(x) = -1 + 2\left(x - \frac{3\pi}{4}\right) = -1 + 2x - \frac{3\pi}{2}$$

$$\boxed{L(x) = -1 - \frac{3\pi}{2} + 2x}$$

5. Find dy for $4y^{3/4} + 3x^{1/2}y - 2x = 0$. Show your work. (8 points)

$$3y^{-1/4}\frac{dy}{dx} + 3x^{1/2}\frac{dy}{dx} + \frac{3}{2}x^{-1/2}y - 2 = 0$$

$$\frac{dy}{dx}\left(3y^{-1/4} + 3x^{1/2}\right) = 2 - \frac{3}{2}x^{-1/2}y.$$

$$\boxed{dy = \left(\frac{2 - \frac{3y}{2\sqrt{x}}}{3y^{-1/4} + 3\sqrt{x}}\right) dx}$$

6. Find dy for $y = e^{5\sqrt{x}+1}$. Show your work. (6 points)

$$\frac{dy}{dx} = e^{5\sqrt{x}+1} \cdot \frac{5}{2\sqrt{x}}$$

$$\boxed{dy = \frac{5}{2}\frac{e^{5\sqrt{x}+1}}{\sqrt{x}}\, dx}$$

7. Find the exact value of the expression. Show your work. (6 points)

$$\tan(\sin^{-1}(\frac{-\sqrt{3}}{2}))$$

$$\tan\left(-\frac{\pi}{3}\right)$$

$$= \boxed{-\sqrt{3}}$$

8. Find the absolute maximum and minimum values of the function on the given interval. Show your work. (8 points)

$$f(x) = \sqrt{-x^2+9}, \quad -2 \le x \le 3$$

$$f'(x) = \frac{-2x}{2\sqrt{-x^2+9}} = \frac{-x}{\sqrt{9-x^2}} = 0$$

$$\Rightarrow \quad x = 0$$

$\underline{x=-2}$

$$f(-2) = \sqrt{-4+9} = \sqrt{5}$$

$\underline{x=3}$

$$f(3) = \sqrt{-9+9} = 0$$

$\underline{x=0}$

$$f(0) = \sqrt{0^2+9} = 3$$

$\therefore$ Absolute maximum $= 3$ at $x = 0$

Absolute minimum $= 0$ at $x = 3$

9. Find the extreme values of the function and where they occur. Show your work. (8 points)

$$y = \frac{7x}{x^2+49}$$

$$\frac{dy}{dx} = \frac{(x^2+49).7 - 7x(2x)}{(x^2+49)^2}$$

$$= \frac{7x^2 + 343 - 14x^2}{(x^2+49)^2}$$

$$= \frac{343-7x^2}{(x^2+49)^2}$$

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{343-7x^2}{(x^2+49)^2} = 0 \Rightarrow 7x^2 = 343$$

$$x^2 = 49 \Rightarrow x = \pm 7$$

$-$ $+$ $-$

-7 7

-8 0 8

$$f'(-8) = \frac{343 - 7\cdot 64}{(64+49)^2} < 0$$

$$f'(0) = \frac{343}{(49)^2} > 0$$

$$f'(8) = \frac{343 - 7\cdot 64}{(64+49)^2} < 0$$

$\therefore x = -7$ is where local minimum is attained.

Local minimum $= f(-7) = \frac{7\cdot -7}{49+49} = \frac{-49}{98} = -\frac{1}{2}$

$\therefore x = 7$ is where local maximum is attained.

Local maximum $= f(7) = \frac{7\cdot 7}{49+49} = \frac{49}{98} = \frac{1}{2}$

10. Answer the following questions about the function whose **derivative** is

$$f'(x) = x^{-3/5}(x-3)$$

(a) What are the critical points of f?

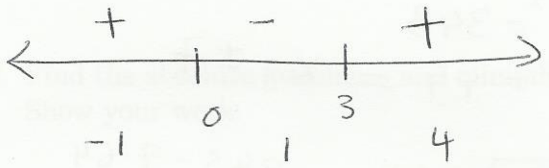
(b) On what intervals is f increasing or decreasing?

(c) At what points, if any, does f assume local maximum and minimum values? Show your work. (8 points)

(a) $f'(x) = \frac{x-3}{x^{3/5}} = 0 \Rightarrow \boxed{x=3}$

Undefined at $\boxed{x=0}$

(b)



$f'(-1) = \frac{-1-3}{(-1)^{3/5}} = \frac{-4}{-1} = 4 > 0$

$f'(1) = \frac{1-3}{1^{3/5}} = \frac{-2}{1} = -2 < 0$

$f'(4) = \frac{4-3}{4^{3/5}} = \frac{1}{4^{3/5}} > 0$

$\therefore$ Increasing: $(-\infty, 0) \cup [3, \infty)$

Decreasing: $(0, 3]$

(c) Local max at $x = 0$.

Local min at $x = 3$.

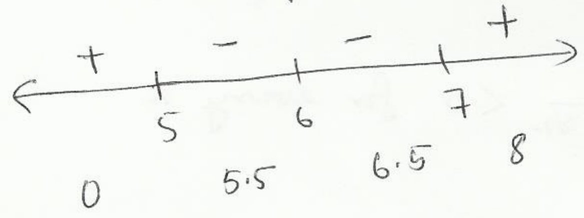
11. Find the intervals on which the function $f(x) = \dfrac{x^2 - 35}{x - 6}$ is increasing and decreasing. Identify the function's local extreme values, if any, saying where they are taken on. Which, if any, of the extreme values are absolute? Show your work. (8 points)

$$f'(x) = \frac{(x-6)2x - (x^2-35)}{(x-6)^2} = \frac{2x^2-12x-x^2+35}{(x-6)^2}$$

$$= \frac{x^2-12x+35}{(x-6)^2} = \frac{(x-7)(x-5)}{(x-6)^2}$$

$$\frac{(x-7)(x-5)}{(x-6)^2} = 0 \Rightarrow x = 7, 5$$

Undefined at $x = 6$



$$f'(0) = \frac{-7 \times -5}{(-6)^2} > 0$$

$$f'(5.5) = \frac{-1.5 \times .5}{(-0.5)^2} < 0$$

$$f'(6.5) = \frac{-0.5 \times 1.5}{(0.5)^2} < 0$$

$$f'(8) = \frac{1.3}{2^2} > 0$$

$\therefore$ Increasing: $(-\infty, 5] \cup [7, \infty)$

Decreasing: $[5, 7]$

Local max is attained at $x=5$. Local max $= f(5) = \frac{25-35}{5-6} = 10$

Local min is attained at $x = 7$, Local min $= f(7) = \frac{49-35}{7-6} = 14$

None of these extreme values are absolute.

12. Graph the function by determining key features of the curve represented by

$$y = \frac{-3x}{\sqrt{x^2+1}}$$

Identify any asymptotes, local and absolute extreme points and inflection points. Show your work. (8 points)

Asymptotes No V.A. , No Oblique asymptotes

$$\lim_{x \to \pm\infty} \frac{-3x}{\sqrt{x^2+1}} = \pm 3 \qquad \therefore\ y = \pm 3 \text{ is a horizontal asymptote}$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+1}\cdot(-3) + 3x \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}}{x^2+1} = \frac{-3(x^2+1)+3x^2}{(x^2+1)^{3/2}} = \frac{-3}{(x^2+1)^{3/2}} = -3(x^2+1)^{-3/2}$$

$$\frac{-3}{(x^2+1)^{3/2}} < 0 \text{ for every } x$$

$\therefore\ y = \frac{-3x}{\sqrt{x^2+1}}$ is always decreasing.

$$\frac{d^2y}{dx^2} = -3\cdot\left(-\frac{3}{\cancel{2}}\right)(x^2+1)^{-5/2}\cdot \cancel{2}x$$

$$= \frac{9x}{(x^2+1)^{5/2}} = 0$$

$\Rightarrow x = 0$

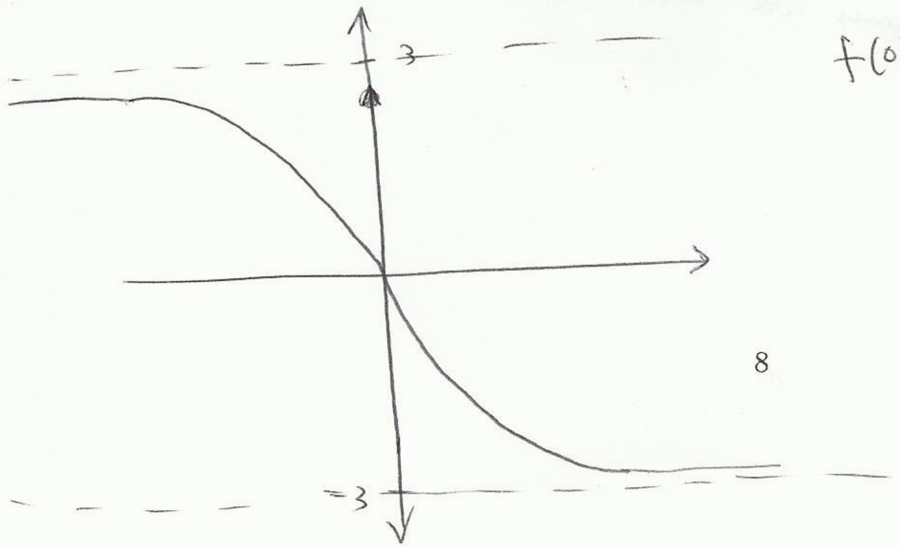
− +

-1 0 1

$$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = \frac{-9}{2^{5/2}} < 0$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = \frac{9}{2^{5/2}} > 0$$

$f(0) = 0$



13. Use L'Hospital's Rule to find the limit. Show your work. (6 points)

$$\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x} \qquad \frac{\infty}{\infty}$$

$$= \lim_{x\to 0^+} \frac{x(2x+2)}{x^2+2x} \qquad \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{4x+2}{2x+2}$$

$$= \boxed{1}$$

14. Find the following limit after identifying its form. Show your work. (6 points)

$$\lim_{x\to 0^+} x^{19}\ln x \qquad 0 \cdot \infty$$

$$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^{19}}} \qquad \frac{\infty}{\infty}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-19}{x^{20}}} = \lim_{x\to 0^+} \frac{x^{19}}{-19} = \boxed{0}$$