Review Problems for Final Exam
MATH 1850, Spring 2013

1. Either $\sin x, \cos x$, or $\tan x$ is given. Find the other two if x lies in the specified interval.
$$\sin x = -\frac{7}{25}, \quad x \in \left[\pi, \frac{3\pi}{2}\right]$$

2. Find the domain and range of the function $g(t) = \sqrt{1 + 9^{-t}}$

3. Simplify using the properties of logarithms.
$$\ln(\cos\theta) - \ln(\frac{\cos\theta}{6})$$

4. Find the limit.
$$\lim_{t\to 5} 9(t-8)(t-4)$$

5. Find the limit.
$$\lim_{h\to 0} \frac{7}{\sqrt{7h+4}+4}$$

6. Find the limit.
$$\lim_{t\to 8} \frac{t^2+3t-88}{t^2-64}$$

7. Use the following function to answer the following questions.
$$f(x) = \begin{cases} x^3, & x \neq 1 \\ \\ -3, & x = -1 \end{cases}$$
(a) Find $\lim_{x\to 1^+} f(x)$, (b) $\lim_{x\to 1^-} f(x)$, (c) Does $\lim_{x\to 4} f(x)$ exist? If so, what is it? If not, why not?

8. Find the limit.
$$\lim_{\theta\to 0} \frac{3\sin\sqrt{4}\theta}{\sqrt{4}\theta}$$

9. Find the limit.
$$\lim_{x\to 0} \frac{\tan 5x}{\sin 6x}$$

10. Determine the point(s) at which the given function $f(x)$ is continuous.
$$f(x) = \frac{11}{x-13} - 6x$$

11. Define $f(4)$ in a way that extends $f(s) = \frac{s^3-64}{s^2-16}$ to be continuous at $s = 4$.

12. Find the limit of the rational function (a) as $x \to \infty$ and (b) as $x \to -\infty$.

$$h(x) = \frac{9x^4}{7x^4 + 11x^3 + 6x^2}$$

13. Find an equation for the tangent to the curve at the given point. Then sketch the curve and the tangent together.

$$y = 8\sqrt{x}, \qquad (1, 8)$$

14. Determine if the following piecewise defined function is differentiable at $x = 0$.

$$f(x) = \begin{cases} 4x - 1, & x \geq 0 \\ x^2 + 3x - 1, & x < 0 \end{cases}$$

What is the right-hand derivative of the given function, $\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$?

15. Using the definition, calculate the derivative of the function. Then find the value of the derivative as specified.

$$g(t) = \frac{5}{t^2}, \qquad g'(-2), g'(3), g'(\sqrt{2})$$

16. Find the derivative of the function.

$$f(s) = \frac{\sqrt{s} - 3}{\sqrt{s} + 1}$$

17. Find all points (x, y) on the graph of $y = \frac{x}{x-1}$ with tangent lines perpendicular to the line $y = x + 3$.

18. An object is dropped from a tower, 175 ft from the ground. The object's height above ground t sec into the fall is $s = 175 - 16t^2$.

    (a) What is the object's velocity, speed, and acceleration at time t?
    (b) About how long does it take the object to hit the ground?
    (c) What is the object's velocity at the moment of impact?

19. Find $\frac{dy}{dx}$ for $y = 9x^2 \sin x + 18x \cos x - 18 \sin x$.

20. Find $\frac{dp}{dq}$ for $p = \frac{\sin q + \cos q}{\cos q}$.

21. Find $\frac{dr}{d\theta}$ for $r = 2 - \theta^5 \sin \theta$.

22. Write the function in the form $y = f(u)$ and $u = g(x)$. Then find $\frac{dy}{dx}$ as a function of x.

$$y = \left(1 - \frac{3x}{4}\right)^{-4}$$

23. Find the derivative of the function below.

$$h(x) = x\cot(2\sqrt{x}) + 19$$

24. Find $\frac{dy}{dt}$.

$$y = (3 + \cos 4t)^{-5}$$

25. Find $\frac{dy}{dt}$.

$$y = \cot^2(\cos^3 t)$$

26. Find the derivative of the given function.

$$y = (x^2 - 5x + 5)e^{6x/5}$$

27. Use implicit differentiation to find $\frac{dy}{dx}$.

$$(6xy + 5)^2 = 12y$$

28. Use implicit differentiation to find $\frac{dy}{dx}$.

$$x\cos(2x + 7y) = y\sin x$$

29. Find the slope of the tangent and normal line to the curve at the given point.

$$x^2y^2 + y = 34, \qquad (3, -2)$$

30. Let $f(x) = 4x^3 - 7x^2 - 2, x \geq 1.5$. Find the value of $\frac{df^{-1}}{dx}$ at the point $x = 323 = f(5)$.

31. Find the derivative of y with respect to x.

$$y = \frac{\ln x}{5 + 3\ln x}$$

32. Find the derivative of y with respect to x.

$$y = (\sin 2x)^{3x}$$

33. Find the derivative of y with respect to x.

$$y = \cos^{-1}(2x^6)$$

34. Find the derivative of y with respect to x.

$$y = \sec^{-1}(5x^2 + 1)$$

35. Assume that all variables are implicit functions of time t. Find $\dfrac{dy}{dt}$.

$$x^2 + 4y^2 + 4y = 17; \qquad \frac{dx}{dt} = 6 \text{ when } x = 4 \text{ and } y = -2$$

36. A metal cube dissolves in acid such that an edge decreases by 0.40 mm/min. How fast is the volume of the cube changing when the edge is 8.20 mm?

37. Find the linearization $L(x)$ of $f(x) = \cot x$ at $x = \dfrac{\pi}{4}$.

38. Find the differential of the given function.

$$y = \frac{9}{5x^2 + 1}$$

39. Find the absolute maximum and minimum values of the function on the given interval.

$$f(x) = \frac{1}{x} + \ln x, \qquad 0.6 \leq x \leq 5$$

40. Determine all critical points for the following function.

$$f(x) = x^2 + \frac{16}{x}$$

41. Sketch the graph of the given function by determining the first and the second derivatives and relevant points.

$$y = x^3 - 7x^2 - 24x + 8$$

42. First use L'Hospital's Rule to evaluate $\lim_{x \to 6} \dfrac{2x - 12}{5x^2 - 180}$. Then determine the limit using limit laws and commonly know limits.

43. Use L'Hospital's Rule to evaluate $\lim_{t \to 0} \dfrac{-2\sin(7t^4)}{-3t}$

44. Find the limit.

$$\lim_{x \to \infty} (1 + 2x)^{11/(2\ln x)}$$

45. Find the limit.

$$\lim_{x\to 1^+} x^{3/(1-x)}$$

46. Use the **lower**, **upper** and **midpoint** rule approximation to estimate the area under the graph of $f(x) = 2x^2$ between $x = 0$ and $x = 10$ with **five** subintervals of equal length.

47. Find the indefinite integral.

$$\int \frac{1}{x^3}\,dx$$

48. Evaluate the following definite integral.

$$\int_1^4 (4x^3 - 2x^2 + 5x - 1)\,dx$$

49. Evaluate the following integral.

$$\int (12x^2 - 4x)\sqrt{4x^3 - 2x^2 + 2}\,dx$$

50. Evaluate the following integral.

$$\int \frac{(\ln x)^{2/3}}{x}\,dx$$

51. Evaluate the following definite integral.

$$\int_0^{\pi/2} \frac{2\sin(2t)}{5 - \cos(2t)}\,dx$$

52. Find the area of the region enclosed by the curves $y = x^2 - 2x$ and $y = -x^2 + 6x$.

53. Find the area of the region enclosed by the curves $y = x^2 - 4$ and $y = 5$.