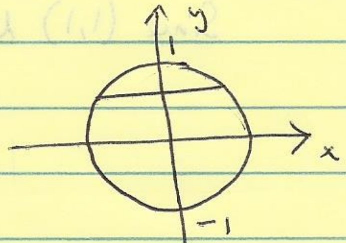
MATH 1930 HW1

Completion points = 8
Selected problem points = 12

6.1 10 Area of cross section $= \frac{1}{2}\left[\sqrt{1-y^2} - \left(-\sqrt{1-y^2}\right)\right]\left[\sqrt{1-y^2} - \left(-\sqrt{1-y^2}\right)\right]$

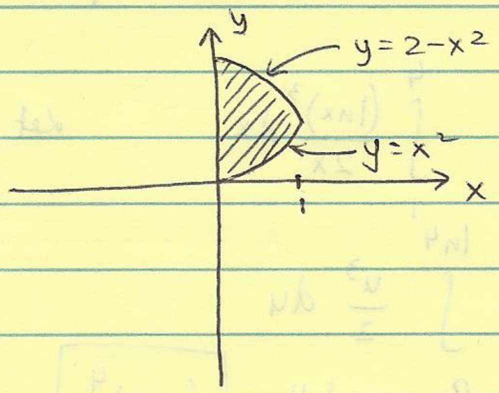
$$= \frac{1}{2}\left(2\sqrt{1-y^2}\right)^2 = 2(1-y^2) = A(y)$$



$$\text{Volume} = \int_{-1}^{1} 2(1-y^2)\,dy = 2\left[y - \frac{y^3}{3}\right]_{-1}^{1}$$

$$= 4\left(1-\frac{1}{3}\right) = \boxed{\frac{8}{3}}$$

6.2 10 $y = 2-x^2$, $y = x^2$, $x = 0$



$2-x^2 = x^2$

$\Rightarrow x = \pm 1$

$$V = \int_0^1 2\pi \left(\text{shell radius}\right)\left(\text{shell height}\right)dx$$

$$= \int_0^1 2\pi\, x\left[(2-x^2)-x^2\right]dx$$

$$= 2\pi \int_0^1 x(2-2x^2)\,dx = 4\pi \int_0^1 (x-x^3)\,dx$$

$$= 4\pi\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= 4\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \boxed{\pi}$$

6.3 19

(a) $L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx \implies \left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$

$\implies \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\implies y = \sqrt{x} + C$

Since (1,1) lie on the curve therefore $1 = \sqrt{1} + C$

$\implies C = 0$

So $y = \sqrt{x}$ from (1,1) to (4,2)

(b) Only one. Because we know the derivative of the function and the value of the function at one value of x.

7.1 11 $\int_1^4 \frac{(\ln x)^3}{2x}\, dx$ Let $u = \ln x$, $du = \frac{1}{x} dx$; $x = 1,\ u = 0$; $x = 4,\ u = \ln 4$

$= \int_0^{\ln 4} \frac{u^3}{2}\, du$

$= \left.\frac{u^4}{8}\right|_0^{\ln 4} = \boxed{\frac{(\ln 4)^4}{8}}$

8.1 | 39 $\int x^3\sqrt{1+x^2}\,dx$ Let $u = 1+x^2$ $x^2 = u-1$

$du = 2x\,dx$

$$= \frac{1}{2}\int x^2\sqrt{u}\,du$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\,du$$

$$= \frac{1}{2}\int u^{3/2}\,du \quad \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{\cancel{2}}\frac{u^{5/2}}{5}\cdot\cancel{2} - \frac{1}{\cancel{2}}\frac{u^{3/2}}{3}\cdot\cancel{2} + C$$

$$= \boxed{\frac{(1+x^2)^{5/2}}{5} - \frac{(1+x^2)^{3/2}}{3} + C}$$

8.2 | 42 $\int 3\sec^4(3x)\,dx$

$$= \int 3\sec^2(3x)\sec^2(3x)\,dx$$

$$= \int 3\left(1+\tan^2(3x)\right)\sec^2(3x)\,dx$$

$$= \int 3\sec^2(3x)\,dx + \int 3\tan^2(3x)\sec^2(3x)\,dx$$

$u = \tan 3x$

$du = 3\sec^2(3x)\,dx$

$$= \tan(3x) + \int 3\tan^2(3x)\sec^2(3x)\,dx$$

$$= \tan(3x) + \int u^2\,du$$

$$= \boxed{\tan(3x) + \frac{\tan^3(3x)}{3} + C}$$