Completion points = 10
Selected problem points = 10

MATH 1930 HW 2

Sec 8.3 16 $\displaystyle\int \frac{x^2\,dx}{4+x^2}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{4\tan^2\theta \cdot 2\sec^2\theta\, d\theta}{4+4\tan^2\theta} = \int \frac{4\tan^2\theta \cdot 2\sec^2\theta}{4\sec^2\theta}\,d\theta$$

$$= \int 2\tan^2\theta\, d\theta$$

$$= 2\int (\sec^2\theta - 1)\,d\theta$$

$$= 2\int \sec^2\theta\, d\theta - 2\int d\theta$$

$$= 2\tan\theta - 2\theta + C$$

$$= \boxed{x - 2\tan^{-1}\left(\frac{x}{2}\right) + C}$$

Sec 8.4 35 $\displaystyle\int \frac{9x^3-3x+1}{x^3-x^2}\,dx$

$$\begin{array}{r|l} & 9 \\ x^3-x^2 & 9x^3+0\cdot x^2-3x+1 \\ & 9x^3-9x^2 \\ \hline & 9x^2-3x+1 \end{array}$$

$$= \int \left(9 + \frac{9x^2-3x+1}{x^3-x^2}\right)dx$$

$$= \int 9\,dx + \int \frac{9x^2-3x+1}{x^3-x^2}\,dx$$

$$= 9x + \int \frac{9x^2-3x+1}{x^2(x-1)}\,dx$$

let $\displaystyle\frac{9x^2-3x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\Rightarrow 9x^2-3x+1 = Ax(x-1) + B(x-1) + Cx^2$

$x=1 \Rightarrow 7 = C$ $\qquad A + C = 9$

$x=0 \Rightarrow 1 = -B \Rightarrow B = -1$ $\qquad \Rightarrow A = 2$

$\therefore \int \frac{9x^3-3x+1}{x^3-x^2}dx = 9x + 2\int \frac{dx}{x} - \int \frac{dx}{x^2} + 7\int \frac{dx}{x-1}$

$= \boxed{9x + 2\ln|x| + \frac{1}{x} + 7\ln|x-1| + C}$

8.7 47 $\int_1^\infty \frac{dx}{1+x^3}$

$0 \le \frac{1}{1+x^3} \le \frac{1}{x^3}$ for $1 \le x \le \infty$

and $\int_1^\infty \frac{1}{x^3}dx$ converges.

$\therefore$ $\boxed{\int_1^\infty \frac{dx}{1+x^3} \text{ also converges.}}$

10.1 22 $a_n = 4n-2, \; n = 1, 2, \ldots$

57 $\lim_{n\to\infty} \left(\frac{3}{n}\right)^{1/n} = \frac{\lim_{n\to\infty} 3^{1/n}}{\lim_{n\to\infty} n^{1/n}} = \frac{1}{1} = 1 \Rightarrow$ converges.