

Completion points = 10

Selected problem points = 10

Homework 3 MATH 1930

10.2 29 $\sum_{n=0}^{\infty} \frac{1}{n+4}$

$$\lim_{n \rightarrow \infty} \frac{1}{n+4} = 0 \Rightarrow \text{Test inconclusive}$$

10.3 29 $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n} = \frac{1}{\ln 2} + \frac{1}{(\ln 2)^2} + \frac{1}{(\ln 2)^3} + \dots$

$$r = \frac{1}{\ln 2} \approx 1.44 > 1 \text{ hence } \boxed{\text{diverges}}$$

10.4 25 $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^n$

$$\frac{n}{3n+1} < \frac{n}{3n} = \frac{1}{3}$$

$$\left(\frac{n}{3n+1} \right)^n < \left(\frac{1}{3} \right)^n$$

↑
Geometric series with $r = \frac{1}{3} < 1$ hence
converges.

$\sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^n$ converges by the Direct Comparison Test.

10.5 21 Ratio Test $a_n = \frac{n^{10}}{10^n} \quad a_{n+1} = \frac{(n+1)^{10}}{10^{n+1}}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^{10} \cdot 10^n}{10^{n+1} \cdot n^{10}} = \left(1 + \frac{1}{n} \right)^{10} \cdot \frac{1}{10}$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{10} \cdot \frac{1}{10} = \frac{1}{10} < 1$$

$\therefore \boxed{\text{Converges}}$

O1 = sum of matched

O2 = limit comparison test

O3 = ratio test

$$\underline{10.6} \quad \underline{16} \quad \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.1)^n}{n}$$

$$|a_n| = \left| (-1)^{n+1} \frac{(0.1)^n}{n} \right| = \frac{(0.1)^n}{n} = \frac{1}{10^n n} < \left(\frac{1}{10}\right)^n$$

convergence test $\rightarrow 0 \uparrow$ pth root

Geometric Series with

$$\sum_{n=0}^{\infty} \left| (-1)^{n+1} \frac{(0.1)^n}{n} \right| \text{ converges.} \quad r = \frac{1}{10} < 1$$

hence

converges absolutely.

does not converge absolutely

$$\frac{(-1)^{n+1}}{n!} = \frac{(-1)^n}{n!} = \frac{(-1)^n}{n!} \cdot \frac{n}{n} = \frac{(-1)^n}{n!} \cdot \frac{n}{n}$$

$$\frac{(-1)^{n+1}}{n!} = \frac{(-1)^{n+1}}{n!} \cdot \frac{n}{n} = \frac{(-1)^{n+1}}{n!} \cdot \frac{n}{n}$$

$$1 > \frac{1}{n} = \frac{1}{n} \left(\frac{1}{n} + 1 \right) \text{ with } \frac{1}{n} < \frac{1}{n!} \text{ and } n > 1$$

