Completion points = 10

Selected problem points = 10

Homework 4 MATH 1930

Sec 10.7

22 $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}3^{2n+2}(x-2)^{n+1}}{\cancel{3}(n+1)}\cdot\frac{\cancel{3}n}{(-1)^n 3^{2n}(x-2)^n}\right|$

$= |x-2|\lim_{n\to\infty}\frac{9n}{n+1} = 9|x-2| < 1$

$\Rightarrow |x-2| < \frac{1}{9}$

$-\frac{1}{9} < x-2 < \frac{1}{9}$

$\frac{17}{9} < x < \frac{19}{9}$

When $x = \frac{17}{9}$ we have $\sum_{n=1}^{\infty}\frac{(-1)^n 3^{2n}}{3n}\left(-\frac{1}{9}\right)^n = \sum_{n=1}^{\infty}\frac{1}{3n}$ divergent

and when $x = \frac{19}{9}$ we have $\sum_{n=1}^{\infty}\frac{(-1)^n 3^{2n}}{3n}\left(\frac{1}{9}\right)^n = \sum_{n=1}^{\infty}\frac{(-1)^n}{3n}$ conditionally convergent.

(a) Radius of convergence $= \frac{1}{9}$, Interval of convergence $= \left(\frac{17}{9}, \frac{19}{9}\right]$

(b) Interval of absolute convergence $= \left(\frac{17}{9}, \frac{19}{9}\right)$

(c) Series converges conditionally at $x = \frac{19}{9}$.

Sec 10.8

3 $f(x) = \ln x$, $f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$, $f'''(x) = \frac{2}{x^3}$

$f(1) = \ln 1 = 0$, $f'(1) = 1$, $f''(1) = -1$, $f'''(1) = 2$

$P_0(x) = 0$, $P_1(x) = (x-1)$, $P_2(x) = (x-1) - \frac{1}{2}(x-1)^2$,

$P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

15 $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\sin 3x = \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)!} = 3x - \frac{3^3 x^3}{3!} + \frac{3^5 x^5}{5!} - \cdots$

10.9 7 $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$

$\ln(1+x^2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x^2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$

$= x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots$

10.10 13 $(1-2x)^3 = 1 + 3(-2x) + \frac{3(2)(-2x)^2}{2!} + \frac{(3)(2)(1)(-2x)^3}{3!}$

$= 1 - 6x + 12x^2 - 8x^3$