MATH 1930 Sec 092

HONORS CALCULUS II

QUIZ 2

February 1, 2013

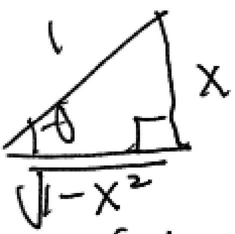
Name (Last, First) Key.

1. Evaluate the integral.

$$\int \frac{(1-x^2)^{1/2}}{x^4}\,dx$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$



$$= \int \frac{\sqrt{1-\sin^2\theta}}{\sin^4\theta}\cos\theta\, d\theta$$

$$= \int \frac{\cos^2\theta}{\sin^4\theta}\,d\theta = \int \frac{1-\sin^2\theta}{\sin^4\theta}\,d\theta$$

$$= \int \frac{1}{\sin^4\theta}\,d\theta - \int \frac{1}{\sin^2\theta}\,d\theta = \int \operatorname{cosec}^4\theta - \int \operatorname{cosec}^2\theta\, d\theta$$

$$= \int \operatorname{cosec}^2\theta \cdot \operatorname{cosec}^2\theta\, d\theta + \cot\theta + C$$

$$= \int \operatorname{cosec}^2\theta\,(1+\cot^2\theta)\,d\theta + \cot\theta + C$$

$$= \int \operatorname{cosec}^2\theta\, d\theta + \int \operatorname{cosec}^2\theta \cot^2\theta\, d\theta + \cot\theta + C$$

$$= -\cancel{\cot}\theta - \frac{\cot^3\theta}{3} + \cancel{\cot\theta} + C$$

$$= -\frac{\cot^3\theta}{3} + C = \boxed{-\frac{(1-x^2)^{3/2}}{3x^3} + C}$$

2. Expand the rational function by partial fractions. You do not have to find the constants.

$$\frac{3x^2-5x+1}{(x^3-3x^2+7x-2)^2(x-1)^3(4x^2-x)}$$

$$= \frac{A_2x^2+A_1x+A_0}{x^3-3x^2+7x-2} + \frac{B_2x^2+B_1x+B_0}{(x^3-3x^2+7x-2)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

$$+ \frac{F_1x+F_0}{4x^2-x}$$