Name (Last, First)_____Key_____

1. Does the following series converge or diverge? Give reasons for your answer.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

Integral Test

$$f(x) = \frac{x}{x^2+1}, \quad f'(x) = \frac{-x^2+1}{(x^2+1)^2} < 0$$

∴ $f(x)$ is decreasing.

$$\int_1^{\infty} \frac{x}{x^2+1} dx = \lim_{b \to \infty} \frac{1}{2} \ln|x^2+1| \Big|_1^b$$

$$= \frac{1}{2} \lim_{b \to \infty} \left[\ln(b^2+1) - \ln 2 \right]$$

$$= \infty$$

∴ $\sum \frac{n}{n^2+1}$ diverges too.

Limit Comparison Test

$$b_n = \frac{1}{n}, \quad a_n = \frac{n}{n^2+1}$$

$$\frac{a_n}{b_n} = \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \frac{n^2}{n^2+1}$$

$$\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{n^2}{n^2+1}$$

$$= \lim_{n \to \infty} \frac{1}{1+\frac{1}{n^2}} = 1 > 0$$

∴ $\sum \frac{n}{n^2+1}$ diverges since $\sum \frac{1}{n}$ diverges

2. Determine if the geometric series converges or diverges. If it converges, then find its sum.

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots + \cdots$$

$$a = 1, \quad r = -\frac{1}{2}$$

$$S = \frac{a}{1-r} = \frac{1}{1-(-\frac{1}{2})} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}$$