MATH 2850 HW1

Completion points = 10
Selected problem points = 10

13.1

11 $r(t) = (2\cos t)i + (3\sin t)j + (4t)k$, $t = \pi/2$

$r'(t) = v(t) = (-2\sin t)i + (3\cos t)j + (4)k$

$r''(t) = a(t) = (-2\cos t)i - (3\sin t)j$

Speed $= |v(t)| = \sqrt{(-2\sin t)^2 + (3\cos t)^2 + 16}$

$|v(\frac{\pi}{2})| = \sqrt{(-2\sin\frac{\pi}{2})^2 + (3\cos\frac{\pi}{2})^2 + 16}$

$= \sqrt{4 + 0 + 16}$

$= \boxed{2\sqrt{5}}$

Direction $= \dfrac{v(\frac{\pi}{2})}{|v(\frac{\pi}{2})|} = \left(-\dfrac{2}{2\sqrt{5}}\sin\frac{\pi}{2}\right)i + \left(\dfrac{3}{2\sqrt{5}}\cos\frac{\pi}{2}\right)j + \dfrac{4}{2\sqrt{5}}k$

$= \boxed{-\frac{1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}k}$

$v(\frac{\pi}{2}) = 2\sqrt{5}\left(-\frac{1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}k\right)$

13.2

21 $V_0 = 500\,m/sec$, $\alpha = 45°$

(a) $t = \dfrac{2V_0\sin\alpha}{g} = \dfrac{2(500)\sin 45°}{9.8} \approx \boxed{72.2\text{ secs}}$

$R = \dfrac{V_0^2}{g}\sin 2\alpha = \dfrac{(500)^2}{9.8}\sin 90° \approx \boxed{25{,}510.2\,m}$

(b) $x = (V_0\cos\alpha)t \Rightarrow 5000m = (500)\cos 45° t \Rightarrow t = \dfrac{5000}{500\cos 45°} \approx 14.4\,sec$

$y = (V_0\sin\alpha)t - \frac{1}{2}gt^2 \Rightarrow y \approx (500)(\sin 45°)(14.14) - \frac{1}{2}(9.8)(14.14)^2$

$= \boxed{4020m}$

(c) $y_{max} = \left(\dfrac{V_0\sin\alpha}{2g}\right)^2 = \dfrac{(500)^2(\sin 45°)^2}{2(9.8)} = \boxed{6378\,m}$

13.3

15 $r(t) = (\sqrt{2}t)i + (\sqrt{2}t)j + (1-t^2)k$

$r'(t) = v(t) = (\sqrt{2})i + (\sqrt{2})j - 2t\,k$

$|v(t)| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2 + (-2t)^2} = \sqrt{4+4t^2} = 2\sqrt{1+t^2}$

$(0,0,1) \Rightarrow t = 0$

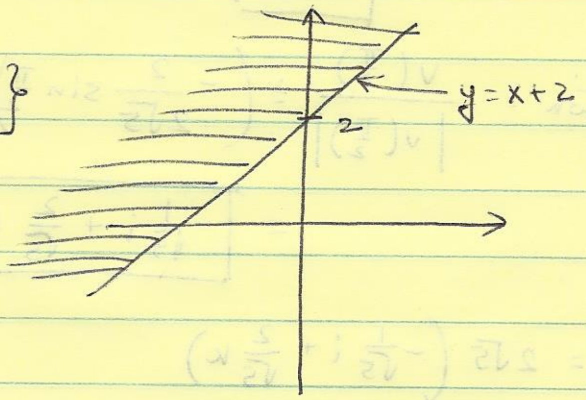
$(\sqrt{2},\sqrt{2},0) \Rightarrow t = 1$

Length $= \int_0^1 2\sqrt{1+t^2}\,dt = \left[2\left(\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ln(t+\sqrt{1+t^2})\right)\right]_0^1 = \boxed{\sqrt{2} + \ln(1+\sqrt{2})}$

14.1

5 $f(x,y) = \sqrt{y-x-2}$

Domain $(f) = \{(x,y) \mid y \geq x+2\}$



14.2

26 $\lim_{(x,y,z)\to(1,-1,-1)} \frac{2xy+yz}{x^2+z^2} = \frac{2(1)(-1)+(-1)(-1)}{1^2+(-1)^2} = \frac{-2+1}{1+1} = \boxed{-\frac{1}{2}}$