Completion points = 10
Selected problem points = 10

MATH 2850 HW 2

14.3

26 $f(x,y,z) = (x^2+y^2+z^2)^{-1/2}$

$f_x(x,y,z) = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2} \cdot \cancel{2}x$

$= \boxed{-x(x^2+y^2+z^2)^{-3/2}}$

$f_y(x,y,z) = \boxed{-y(x^2+y^2+z^2)^{-3/2}}$

$f_z(x,y,z) = \boxed{-z(x^2+y^2+z^2)^{-3/2}}$

53 $w = xy^2 + x^2y^3 + x^3y^4$

$w_x = y^2 + 2xy^3 + 3x^2y^4$

$w_{xy} = 2y + 6xy^2 + 12x^2y^3$

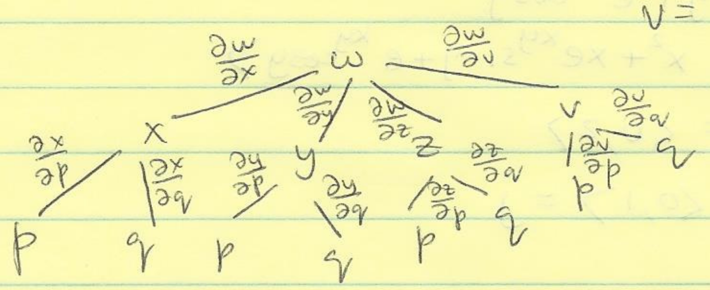
$w_y = 2xy + 3x^2y^2 + 4x^3y^3$

$w_{yx} = 2y + 6xy^2 + 12x^2y^3$

$\therefore \boxed{w_{xy} = w_{yx}}$

14.4

22 $w = f(x,y,z,v)$, $x = g(p,q)$, $y = h(p,q)$, $z = j(p,q)$, $v = k(p,q)$



$$\boxed{\frac{\partial w}{\partial p} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial p} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial p} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial p} + \frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial p}}$$

14.5 13 $g(x,y) = \frac{x-y}{xy+2}$

$$g_x(x,y) = \frac{(xy+2)(1) - (x-y)y}{(xy+2)^2} = \frac{y^2+2}{(xy+2)^2}$$

$$g_x(1,-1) = \frac{1^2+2}{(-1+2)^2} = 3$$

$$g_y(x,y) = \frac{(xy+2)(-1) - (x-y)x}{(xy+2)^2} = -\frac{x^2+2}{(xy+2)^2}$$

$$g_y(1,-1) = -\frac{1^2+2}{(-1+2)^2} = -3$$

$\nabla g = \langle 3, -3 \rangle$

Unit vector along $12i+5j = \frac{12i+5j}{\sqrt{12^2+5^2}} = \langle \frac{12}{13}, \frac{5}{13} \rangle$

$$\therefore (D_u g)_{P_0} = \nabla g \cdot \langle \tfrac{12}{13}, \tfrac{5}{13} \rangle = \langle 3,-3 \rangle \cdot \langle \tfrac{12}{13}, \tfrac{5}{13} \rangle$$
$$= \frac{36}{13} - \frac{15}{13} = \boxed{\frac{21}{13}}$$

20 $f(x,y,z) = x^2y + e^{xy}\sin y, \quad P_0(1,0)$

$f_x(x,y,z) = 2xy + ye^{xy}\sin y$

$f_y(x,y,z) = x^2 + xe^{xy}\sin y + e^{xy}\cos y$

$\nabla f = \langle 2xy + ye^{xy}\sin y,\ x^2 + xe^{xy}\sin y + e^{xy}\cos y \rangle$

$\nabla f(1,0) = \langle 0, 1^2+1 \rangle = \langle 0, 2 \rangle$

$$u = \frac{\nabla f(1,0)}{|\nabla f(1,0)|} = \frac{\langle 0,2 \rangle}{2} = \langle 0,1 \rangle = j$$

f increases most in the direction of $u = j$ and decreases most in the direction of $-u = -j$

$(D_u f)_{P_0} = \nabla f \cdot u = \langle 0,2 \rangle \cdot \langle 0,1 \rangle = 2$

$(D_{-u} f)_{P_0} = \nabla f \cdot (-u) = \langle 0,2 \rangle \cdot \langle 0,-1 \rangle = -2$