Completion points = 10
Selected problem points = 10

MATH 2850 HW3

14.6

20 $\nabla f = \langle e^x \cos yz, -ze^x \sin yz, -ye^x \sin yz \rangle$

$\nabla f(0,0,0) = \langle 1, 0, 0 \rangle$

$u = \frac{v}{|v|} = \frac{2i + 2j - 2k}{\sqrt{2^2 + 2^2 + (-2)^2}} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle$

$\therefore df = (\nabla f \cdot u)\, ds = \left(\langle 1,0,0 \rangle \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle \right) 0.1$

$= \frac{1}{\sqrt{3}} \times 0.1 = \boxed{0.0577}$

27 (a) $f(x,y) = 3x - 4y + 5$ $\quad f(0,0) = 5$

$f_x(x,y) = 3 \quad , \quad f_y(x,y) = -4$

$f_x(0,0) = 3, \quad f_y(0,0) = -4$

$\therefore L(x,y) = 5 + 3(x-0) - 4(y-0) = \boxed{3x - 4y + 5}$

(b) $f_x(1,1) = 3, \quad f_y(1,1) = -4 \quad , \quad f(1,1) = 4$

$\therefore L(x,y) = 4 + 3(x-1) - 4(y-1) = \boxed{3x - 4y + 5}$

14.7

2 $f(x,y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$

$f_x(x,y) = 2y - 10x + 4 = 0 \quad f_{xx}(x,y) = -10 \quad , \quad f_{xy}(x,y) = 2$

$f_y(x,y) = 2x - 4y + 4 = 0 \quad f_{yy}(x,y) = -4$

Solving gives $x = \frac{2}{3}, y = \frac{4}{3}$ Critical point is $(\frac{2}{3}, \frac{4}{3})$

$f_{xx}(\frac{2}{3}, \frac{4}{3}) \cdot f_{yy}(\frac{2}{3}, \frac{4}{3}) - f_{xy}(\frac{2}{3}, \frac{4}{3})^2 = -10 \cdot -4 - 2^2$

$= 40 - 4 = 36 > 0$ and $f_{xx}(\frac{2}{3}, \frac{4}{3}) < 0$

$\therefore$ Local maximum at $(\frac{2}{3}, \frac{4}{3}) = f(\frac{2}{3}, \frac{4}{3}) = 0$

25 $f(x,y) = e^{x^2+y^2-4x}$

$f_x(x,y) = (2x-4)\,e^{x^2+y^2-4x} = 0$, $f_{xx}(x,y) = (2x-4)^2 e^{x^2+y^2-4x} + 2e^{x^2+y^2-4x}$

$f_y(x,y) = 2y\,e^{x^2+y^2-4x} = 0$

$\Rightarrow x = 2,\ y = 0$ $\quad f_{yy}(x,y) = 4y^2 e^{x^2+y^2-4x} + 2e^{x^2+y^2-4x}$

Critical point is $(2,0)$

$f_{xy}(x,y) = (2x-4)\,2y\,e^{x^2+y^2-4x}$

$$f_{xx}(2,0) = \frac{2}{e^4},\quad f_{yy}(2,0) = \frac{2}{e^4},\quad f_{xy}(2,0) = 0$$

$$\therefore\ f_{xx}(2,0)\,f_{yy}(2,0) - f_{xy}(2,0)^2 = \frac{4}{e^8} > 0 \text{ and } f_{xx}(2,0) > 0$$

$$\therefore\ \boxed{\text{Local minimum at } (2,0) = f(2,0) = \frac{1}{e^4}}$$

14.8 14 $f(x,y) = 3x - y + 6,\quad g(x,y) = x^2+y^2-4 = 0$

$\nabla f = \langle 3,-1\rangle,\quad \nabla g = \langle 2x, 2y\rangle$

$\nabla f = \lambda \nabla g \Rightarrow \langle 3,-1\rangle = \langle 2x\lambda, 2y\lambda\rangle$

$\Rightarrow 2x\lambda = 3,\quad 2y\lambda = -1$

$x = \frac{3}{2\lambda}\quad y = -\frac{1}{2\lambda}$

$x^2+y^2 = 4 \Rightarrow \frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} = 4 \Rightarrow \frac{10}{4\lambda^2} = 4 \quad \lambda^2 = \frac{10}{16}$

$\lambda = \pm\sqrt{\frac{5}{8}} = \pm\frac{\sqrt{5}}{2\sqrt{2}}$

$$\therefore\ x = \frac{3}{2\lambda} = \pm\frac{3\sqrt{2}}{\sqrt{5}} = \pm\frac{6}{\sqrt{10}}$$

$$y = -\frac{1}{2\lambda} = \mp\frac{\sqrt{2}}{\sqrt{5}} = \mp\frac{2}{\sqrt{10}}$$

Points are $(x,y) = \left(\frac{6}{\sqrt{10}}, -\frac{2}{\sqrt{10}}\right)$ and $\left(-\frac{6}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right)$

$f\left(\frac{6}{\sqrt{10}}, -\frac{2}{\sqrt{10}}\right) = \frac{20}{\sqrt{10}} + 6 = 2\sqrt{10} + 6 = 12.325$ maximum value.

$f\left(\frac{-6}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right) = -2\sqrt{10} + 6 = -0.325$ minimum value.