Selected problem points = 10
Completion points = 10

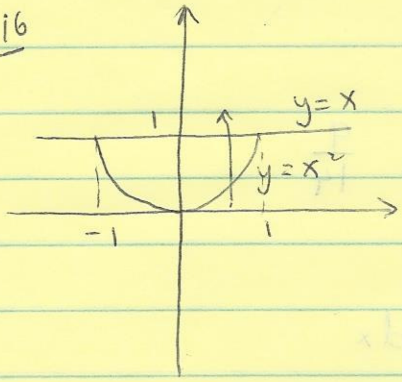
MATH 2850
Homework 5

15.5 7

$$\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)\,dz\,dy\,dx = \int_0^1\int_0^1 \left(x^2+y^2+\tfrac{1}{3}\right)dy\,dx$$

$$= \int_0^1 \left(x^2+\tfrac{2}{3}\right)dx = \tfrac{1}{3}+\tfrac{2}{3} = \boxed{1}$$

15.6 16



$\delta(x,y) = y+1$

$$M = \int_{-1}^{1}\int_{x^2}^{1} (y+1)\,dy\,dx$$

$$= \int_{-1}^{1} \frac{y^2}{2}+y\Big|_{x^2}^{1} dx$$

$$= \int_{-1}^{1} \left(\frac{1}{2}+1\right) - \frac{x^4}{2} - x^2\,dx$$

$$= -\int_{-1}^{1}\left(\frac{x^4}{2}+x^2-\frac{3}{2}\right)dx = -\left[\frac{x^5}{10}+\frac{x^3}{3}-\frac{3x}{2}\right]_{-1}^{1}$$

$$= \boxed{\frac{32}{15}}$$

$$M_x = \int_{-1}^{1}\int_{x^2}^{1} y(y+1)\,dy\,dx$$

$$= \int_{-1}^{1} \frac{y^3}{3}+\frac{y^2}{2}\Big|_{x^2}^{1} dx$$

$$= \int_{-1}^{1} \left(\frac{1}{3}+\frac{1}{2}\right) - \left(\frac{x^6}{3}+\frac{x^4}{2}\right)dx = -\int_{-1}^{1}\left(\frac{x^6}{3}+\frac{x^4}{2}-\frac{5}{6}\right)dx$$

$$= -\left[\frac{x^7}{21}+\frac{x^5}{10}-\frac{5x}{6}\right]_{-1}^{1} = \boxed{\frac{48}{35}}$$

$$M_y = \int_{-1}^{1}\int_{x^2}^{1} x(y+1)\,dy\,dx = \int_{-1}^{1} \frac{xy^2}{2} + xy \Big|_{x^2}^{1} dx = \int_{-1}^{1}\left(\frac{x}{2}+x\right)-\left(\frac{x^5}{2}+x^3\right)dx$$

$$= \int_{-1}^{1}\left(\frac{3x}{2} - \frac{x^5}{2} - x^3\right)dx$$

$$= \frac{3x^2}{4} - \frac{x^6}{12} - \frac{x^4}{4}\Big|_{-1}^{1}$$

$$= 0$$

Center of mass $(\bar{x}, \bar{y})$ $\quad \bar{x} = \frac{M_y}{M} = 0$

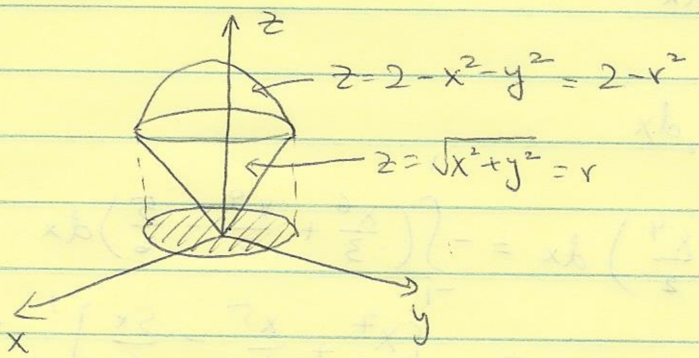
$$\bar{y} = \frac{M_x}{M} = \frac{\frac{48}{35}}{\frac{32}{15}} = \frac{9}{14}$$

$$I_y = \int_{-1}^{1}\int_{x^2}^{1} x^2(y+1)\,dy\,dx = \int_{-1}^{1} \frac{x^2y^2}{2} + x^2y \Big|_{x^2}^{1} dx$$

$$= \int_{-1}^{1}\left(\frac{x^2}{2}+x^2\right) - \left(\frac{x^6}{2}+x^4\right)dx$$

$$= \int_{-1}^{1}\left(\frac{3x^2}{2} - \frac{x^6}{2} - x^4\right)dx = \boxed{\frac{16}{35}}$$

15.7 12



(a) $\int_0^{2\pi}\int_0^{1}\int_r^{2-r^2} dz\, r\,dr\,d\theta$ $\quad$ (b) $\int_0^{2\pi}\int_0^{1}\int_0^{z} r\,dr\,dz\,d\theta + \int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{2-z}} r\,dr\,dz\,d\theta$

(c) $\int_0^{1}\int_r^{2-r^2}\int_0^{2\pi} r\,d\theta\,dz\,dr$

16.1 7 $r = (t^2-1)\hat{j} + 2t\hat{k} \Rightarrow y = t^2-1,\ z = 2t$

$$\therefore \frac{z^2}{4} - 1 = y \Rightarrow (f).$$

16.2 19 $r = t\hat{i} + t^2\hat{j} + tk,\ 0 \le t \le 1.$

$$F = xy\hat{i} + y\hat{j} - yz\hat{k}$$

$$\Rightarrow F(r(t)) = t^3\hat{i} + t^2\hat{j} - t^3\hat{k}$$

$$\frac{dr}{dt} = \hat{i} + 2t\hat{j} + \hat{k}$$

$$F \cdot \frac{dr}{dt} = \langle t^3, t^2, -t^3\rangle \cdot \langle 1, 2t, 1\rangle = t^3 + 2t^3 - t^3 = 2t^3$$

$$\text{Work} = \int_0^1 2t^3\,dt = \boxed{\frac{1}{2}}$$