Completion points = 10
Selected problem points = 10

HW 6
MATH 2850

Sec 16.4

6 $M = x^2+4y, \quad N = x+y^2 \Rightarrow \frac{\partial M}{\partial x} = 2x, \ \frac{\partial M}{\partial y} = 4, \ \frac{\partial N}{\partial x} = 1, \ \frac{\partial N}{\partial y} = 2y.$

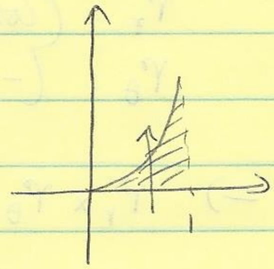
$$\therefore \text{Flux} = \iint_R (2x+2y)\,dx\,dy = \int_0^1\int_0^1 (2x+2y)\,dx\,dy.$$

$$= \int_0^1 x^2+2xy\Big|_0^1 dy$$

$$= \int_0^1 (1+2y)\,dy = y+y^2\Big|_0^1 = \boxed{2}$$

$$\text{Circulation} = \iint_R (1-4)\,dx\,dy = \int_0^1\int_0^1 -3\,dx\,dy = \boxed{-3}$$

19 $M = 2xy^3, \ N = 4x^2y^2 \Rightarrow \frac{\partial M}{\partial y} = 6xy^2; \ \frac{\partial N}{\partial x} = 8xy^2$



$0 \le y \le x^3$

$0 \le x \le 1$

$$\text{Work} = \oint_C 2xy^3\,dx + 4x^2y^2\,dy$$

$$= \iint_R (8xy^2 - 6xy^2)\,dx\,dy.$$

$$= \int_0^1\int_0^{x^3} 2xy^2\,dy\,dx$$

$$= \int_0^1 \frac{2xy^3}{3}\Big|_0^{x^3} dx = \frac{2}{3}\int_0^1 x^{10}\,dx = \frac{2}{33}x^{11}\Big|_0^1 = \boxed{\frac{2}{33}}$$

Sec 16.5 3 $x = r\cos\theta, \quad y = r\sin\theta, \quad z = \frac{\sqrt{x^2+y^2}}{2} = \frac{r}{2}$

Since $0 \le z \le 3 \Rightarrow 0 \le \frac{r}{2} \le 3 \Rightarrow 0 \le r \le 6$

$\therefore \mathbf{r}(r,\theta) = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j} + \left(\frac{r}{2}\right)\hat{k} \qquad 0 \le r \le 6$
$0 \le \theta \le \frac{\pi}{2}$

19 $x = r\cos\theta, \quad y = r\sin\theta \qquad z = 2\sqrt{x^2+y^2} = 2r$

Since $2 \le z \le 6 \quad \therefore 2 \le 2r \le 6$
$\Rightarrow 1 \le r \le 3 \qquad 0 \le \theta \le 2\pi$

$\therefore \mathbf{r}(r,\theta) = (r\cos\theta)i + (r\sin\theta)j + 2r\,k$
$\mathbf{r}_r = (\cos\theta)i + (\sin\theta)j + 2k$
$\mathbf{r}_\theta = (-r\sin\theta)i + (r\cos\theta)j$

$$\Rightarrow \mathbf{r}_r \times \mathbf{r}_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 2 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = (-2r\cos\theta)i - (2r\sin\theta)j + (r\cos^2\theta + r\sin^2\theta)k$$
$$= (-2r\cos\theta)i - (2r\sin\theta)j + rk$$

$\therefore |\mathbf{r}_r \times \mathbf{r}_\theta| = \sqrt{4r^2\cos^2\theta + 4r^2\sin^2\theta + r^2} = \sqrt{5r^2} = r\sqrt{5}$

$$\therefore A = \int_0^{2\pi}\int_1^3 r\sqrt{5}\,dr\,d\theta = \int_0^{2\pi} \frac{r^2\sqrt{5}}{2}\Big|_1^3 d\theta = \int_0^{2\pi} 4\sqrt{5}\,d\theta = \boxed{8\pi\sqrt{5}}$$

42 $\quad p = k, \quad \nabla f = 2xi + 2yj + 2zk$

$$|\nabla f| = \sqrt{4x^2+4y^2+4z^2} = \sqrt{8} = 2\sqrt{2}$$

$$|\nabla f \cdot p| = 2z$$

$x^2+y^2+z^2 = 2$ and $z = \sqrt{x^2+y^2} \Rightarrow x^2+y^2+x^2+y^2 = 2$

$\Rightarrow \quad x^2+y^2 = 1$

$$\therefore S = \iint_R \frac{|\nabla f|}{|\nabla f \cdot p|}\, dA = \iint_R \frac{\cancel{2}\sqrt{2}}{\cancel{2}z}\, dA$$

$$= \sqrt{2}\iint_R \frac{1}{z}\, dA$$

$$= \sqrt{2}\iint_R \frac{1}{\sqrt{2-(x^2+y^2)}}\, dA = \sqrt{2}\int_0^{2\pi}\int_0^1 \frac{r\,dr\,d\theta}{\sqrt{2-r^2}}$$

$$= \sqrt{2}\int_0^{2\pi} (-1+\sqrt{2})\, d\theta$$

$$= \boxed{2\pi(2-\sqrt{2})}$$

Sec 16.6 $\quad$ 2 $\quad r(x,y) = x\hat{i} + y\hat{j} + \sqrt{4-y^2}\,\hat{k}, \quad -2 \le y \le 2, \quad r_x = \hat{i}$

$$r_y = \hat{j} - \frac{y}{\sqrt{4-y^2}}\hat{k}$$

$$\Rightarrow r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -\frac{y}{\sqrt{4-y^2}} \end{vmatrix} = \frac{y}{\sqrt{4-y^2}}\hat{j} + \hat{k}$$

$$|r_x \times r_y| = \sqrt{\frac{y^2}{4-y^2}+1} = \frac{2}{\sqrt{4-y^2}}$$

$$\therefore \iint_S G(x,y,z)\, d\sigma = \int_1^4\int_{-2}^2 \sqrt{\cancel{4-y^2}}\left(\frac{2}{\sqrt{\cancel{4-y^2}}}\right) dy\, dx = 2\cdot 3\cdot 4 = \boxed{24}$$