Name (Last, First)___*Key*_____

1. $\mathbf{r}(t)$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. Then find the particle's speed.

$$\mathbf{r}(t) = (1+t)\mathbf{i} + \frac{t^2}{\sqrt{2}}\mathbf{j} + \frac{t^3}{3}\mathbf{k}$$

$\mathbf{v}(t) = \mathbf{r}'(t) = \mathbf{i} + \frac{2t}{\sqrt{2}}\mathbf{j} + \frac{3t^2}{3}\mathbf{k}$

$= \boxed{\mathbf{i} + \sqrt{2}t\,\mathbf{j} + t^2\mathbf{k}}$

$\mathbf{a}(t) = \mathbf{v}'(t) = \boxed{\sqrt{2}\,\mathbf{j} + 2t\,\mathbf{k}}$

$\text{Speed} = |\mathbf{v}(t)| = \sqrt{1^2 + 2t^2 + t^4} = \sqrt{(1+t^2)^2} = \boxed{1+t^2}$

2. Evaluate the integral.

$$\int_1^4 \left[\frac{1}{t}\mathbf{i} + \frac{1}{5-t}\mathbf{j} + \frac{1}{2t}\mathbf{k}\right] dt$$

$= \int_1^4 \frac{1}{t}\mathbf{i}\,dt + \int_1^4 \frac{1}{5-t}\mathbf{j}\,dt + \int_1^4 \frac{1}{2t}\mathbf{k}\,dt$

$= \ln|t|\mathbf{i}\Big|_1^4 - \ln|5-t|\Big|_1^4\mathbf{j} + \frac{1}{2}\ln t\Big|_1^4\mathbf{k}$

$= (\ln 4)\mathbf{i} - (\ln 1)\mathbf{i} - (\ln 1)\mathbf{j} + (\ln 4)\mathbf{j} + \frac{1}{2}(\ln 4)\mathbf{k} - \frac{1}{2}(\ln 1)\mathbf{k}$

$= \boxed{(\ln 4)\mathbf{i} + (\ln 4)\mathbf{j} + (\ln 2)\mathbf{k}}$