

MATH 2850 Sec 003
ELEMENTARY MULTIVARIABLE CALCULUS

QUIZ 1
January 16, 2013

Name (Last, First) Key

1. $\mathbf{r}(t)$ is the position of a particle in space at time t . Find the particle's velocity and acceleration vectors. Then find the particle's speed.

$$\mathbf{r}(t) = (1+t)\mathbf{i} + \frac{t^2}{\sqrt{2}}\mathbf{j} + \frac{t^3}{3}\mathbf{k}$$

$$\mathbf{v}(t) = \mathbf{r}'(t) = \mathbf{i} + \frac{2t}{\sqrt{2}}\mathbf{j} + \frac{3t^2}{3}\mathbf{k}$$

$$= \boxed{\mathbf{i} + \sqrt{2}t\mathbf{j} + t^2\mathbf{k}}$$

$$\mathbf{a}(t) = \mathbf{v}'(t) = \boxed{\sqrt{2}\mathbf{j} + 2t\mathbf{k}}$$

$$\text{Speed} = |\mathbf{v}(t)| = \sqrt{1^2 + 2t^2 + t^4} = \sqrt{(1+t^2)^2} = \boxed{1+t^2}$$

2. Evaluate the integral.

$$\begin{aligned} & \int_1^4 \left[\frac{1}{t}\mathbf{i} + \frac{1}{5-t}\mathbf{j} + \frac{1}{2t}\mathbf{k} \right] dt \\ &= \int_1^4 \frac{1}{t}\mathbf{i} dt + \int_1^4 \frac{1}{5-t}\mathbf{j} dt + \int_1^4 \frac{1}{2t}\mathbf{k} dt \\ &= \left. \ln|t|\mathbf{i} \right|_1^4 - \left. \ln|5-t|\mathbf{j} \right|_1^4 + \left. \frac{1}{2}\ln t\mathbf{k} \right|_1^4 \\ &= (\ln 4)\mathbf{i} - (\ln 1)\mathbf{i} - (\ln 1)\mathbf{j} + (\ln 4)\mathbf{j} + \frac{1}{2}(\ln 4)\mathbf{k} - \frac{1}{2}(\ln 1)\mathbf{k} \\ &= \boxed{(\ln 4)\mathbf{i} + (\ln 4)\mathbf{j} + (\ln 2)\mathbf{k}} \end{aligned}$$