MATH 2850 Sec 003
ELEMENTARY MULTIVARIABLE CALCULUS
QUIZ 6

April 10, 2013

Name (Last, First) Key

1. Construct the integral to find the work done by **F** over the curve in the direction of increasing t. You **do not** have to evaluate the integral. Show your work.

$$\mathbf{F} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$$

$$\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 2\pi$$

$$F(r(t)) = t\,i + \sin t\,j + \cos t\,k$$

$$\frac{dr}{dt} = \cos t\,i - \sin t\,j + k$$

$$F(r(t)) \cdot \frac{dr}{dt} = t\cos t - \sin^2 t + \cos t$$

$$\boxed{W = \int_0^{2\pi} (t\cos t - \sin^2 t + \cos t)\,dt}$$

2. Find the divergence(div) and circulation density(circ. density) of the vector field

$$\mathbf{F} = e^x y\mathbf{i} + (x - y^2)\mathbf{j}$$

(Hint: For a vector field $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$, div $\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$ and circ. density $\mathbf{F} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$)

$$\frac{\partial M}{\partial x} = e^x y, \quad \frac{\partial N}{\partial y} = -2y$$

$$\frac{\partial M}{\partial y} = e^x y, \quad \frac{\partial N}{\partial x} = 1$$

$$\boxed{\text{div}\,F = e^x y - 2y} \qquad \boxed{\text{circ. density}\,F = 1 - e^x y}$$