

Math 1260 Midterm Exam #1 - May 25, 2006

Directions: Place all work you wish to be graded in the blue book, you may keep the actual exam. Scientific calculators are allowed. Please clearly label the problems in the blue book, especially if you do them out of order. Also clearly cross out any work you do not wish to have graded.

1. **(15 points)** Solve the following inequality. Give your answer in interval notation *and* graph the solution on the number line.

$$\frac{x^2 + x}{x^2 - 1} > 3.$$

2. **(15 points)** Write the expression as a single fraction in lowest terms:

$$\frac{2y}{3y^2 - 11y - 4} - \frac{y}{3y^2 + 7y + 2}$$

3. **(15 points)** I deposit \$10,000 into a bank account earning 5.5% annual interest rate compounded weekly. (52 times per year). How much will I have after three and a half years?

4. **(15 points)** A bank account pays an annual rate of 4.5% compounded daily (365 times/year).

a. Find the APY. (a.k.a. effective rate).

b. Do you think it is possible for the bank to compound frequently enough (e.g. hourly, minutely, every second...) to make the effective rate more than 4.7%? Explain.

5. **(20 points)**

- a. Find the monthly payment on a 20-year, \$150,000 mortgage with a 5.9% annual rate.
- b. Determine that total amount of interest paid to the bank during the 20 years (240 total payments).
- c. How much of the first month's payment went to principal? How much to interest?

6. **(20 points)** Suppose Joe deposits \$1000 at the end of each year for 8 years in an account earning 6% compounded annually. I am going to deposit a lump sum today into an account earning 5% compounded semiannually. How much do I need to deposit today so that after 8 years, Joe and I have the same amount of money?

Exam Solutions

1. First solve equality: $\frac{x^2+x}{x^2-1} = 3$

$$x^2+x=3x^2-3$$

$$2x^2-x-3=0$$

$$(2x-3)(x+1)=0$$

$$x = \frac{3}{2} \quad x = -1$$

Also set denom = 0

$$x^2-1=0 \Rightarrow x = 1, -1$$

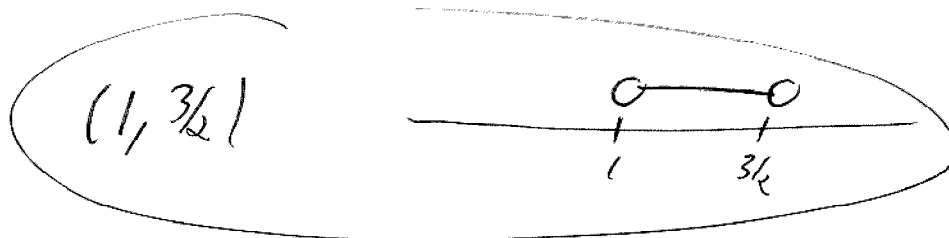


Test -2 $\frac{4-2}{4-1} = \frac{2}{3}$ NO

0 $\frac{0}{1} = 0$ NO

1.25 $\frac{2.875}{.5625} > 3$ YES

2 $\frac{6}{3} = 2$ NO



$(1, \frac{3}{2})$

$$2. \quad \frac{2y}{(3y+1)(y-4)} - \frac{y}{(3y+1)(y+2)} = \frac{2y(y+2) - y(y-4)}{(3y+1)(y-4)(y+2)}$$

$$= \frac{2y^2 + 4y - y^2 + 4y}{(3y+1)(y-4)(y+2)} = \frac{y^2 + 8y}{(3y+1)(y-4)(y+2)}$$

$$3. \quad 10000 \left(1 + \frac{.055}{52}\right)^{52 \cdot 3.5}$$

$$= 10000 (1.00105769)^{182} = \$12121.53$$

$$4. \quad a. \quad r_e = \left(1 + \frac{.045}{365}\right)^{365} - 1 = .04602 \text{ or } 4.602\%$$

b. compounding every minute now gives

$$r_e = \left(1 + \frac{.045}{365.24}\right)^{365.24} - 1 = .0460277$$

every minute

$$r_e = \left(1 + \frac{.045}{365.24.60}\right)^{365.24.60} - 1 = .0460278$$

It appears to be converging to a rate which is less than 4.7%, so NO

5.

$$a. \quad 150000 = R \left[\frac{1 - \left(1 + \frac{.059}{12}\right)^{-240}}{\frac{.059}{12}} \right]$$

$$= R (140.71149) \quad R = \$1066.01$$

$$b \quad \text{Total payments is } 240 \cdot (1066.01) = 255,842.63$$

$$\text{Total interest is } \$105,842.63$$

c First month's interest is

$$\left(\frac{.059}{12}\right) (150000) = \$737.50$$

Thus first month's principal is

$$1066.01 - 737.50 = 328.51$$

6. Joe $S = 1000 \left(\frac{(1.06)^8 - 1}{.06} \right) = \9897.47

me $9897.47 = P(1.025)^{16}$
 $= P \cdot 1.4845$

$$P = \$6667.18$$

I must deposit $\$6667.18$