

7004/110110

Math 1260 Midterm Exam #2 - June 8, 2006

Instructions: Scientific calculators may be used, graphing calculators are forbidden.

1. (10 points) The half-life of radium 226 is approximately 1620 years.
- How much of a sample weighing 2 grams will remain after 100 years?
 - How much time is necessary for a sample weighing 2 grams to decay to 0.1 grams?

First find k :

$$y = y_0 e^{kt}$$

$$\frac{1}{2}y_0 = y_0 e^{1620k}$$

$$\frac{1}{2} = e^{1620k}$$

$$k = \frac{\ln(\frac{1}{2})}{1620} = -0.00042787$$

a. $y = 2 e^{-0.00042787 \cdot 100} = 1.916 \text{ grams}$

b. $0.1 = 2 e^{-0.00042787 t}$

$$0.05 = e^{-0.00042787 t}$$

$$\ln(0.05) = -0.00042787 t$$

$$t = 700.5 \text{ years}$$

2. (12 points) Evaluate:

a. $\log_3(81) = 4$

b. $a^{2\log_a(10)} = 100$

c. $\ln(e^5) = 5$

d. $\log_{10}(0.001) = -3$

e. $\log_a(a^{19}) = 19$

f. $\log_{1/3}(9) = -2$

3. (10 points) Solve for x , rounding your answer to the nearest 100th.

a. $2e^{5x+12} = 8$

$$e^{5x+12} = 4$$

$$5x+12 = \ln 4$$

$$x = \frac{\ln 4 - 12}{5} \approx -2.12$$

b. $\log_4 x - \log_4(x+6) = 2$

$$\frac{x}{x+6} = 16$$

$$16x + 96 = x$$

$$4 \log_4 x - \log_4(x+6) = 16$$

$$4 \log_4 \left(\frac{x}{x+6} \right) = 16$$

$$15x = -96$$

$$x = -6.4$$

but no solution
 \downarrow $\log_4 x$ not \downarrow

4. (8 points) How much does \$2000 grow to if invested at an annual rate of 7.3% compounded continuously for 6 years.

$$2000 e^{.073 \cdot 6} = 3099.21$$

5. (10 points)

a. Give a function $f(x)$ so that the graph of $y = f(x)$ has vertical asymptotes $x = 2$ and $x = 3$.

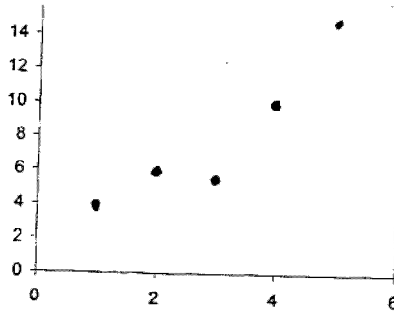
b. Give a function $g(x)$ so that the graph of $y = g(x)$ has a horizontal asymptote $y = 3$. For extra credit find a single function that is simultaneously solution to both parts of the problem.

$$f(x) = \frac{3x^2}{(x-2)(x-3)}$$

6. (8 points) An account pays 4.9% annual rate compounded continuously. What is the effective rate (APY)?

$$r_p = e^{.049} - 1 = .05022$$

7. (6 points) Below is a scatterplot of 5 data points. Which is the best approximation to the correlation coefficient?



- a. $r = 0.93$ b. $r = -0.93$ c. $r = 0.07$ d. $r = -0.07$ e. $r = 0$.

8. (10 points) Let $A = (2, 1)$, $B = (5, 0)$, $C = (3, 6)$ be three points in the $x-y$ plane. Use slopes to determine if ABC is a right triangle, and if so determine which angle is 90 degrees.

$$\text{slope } \overrightarrow{AB} = \frac{0-1}{5-2} = -\frac{1}{3}$$

$$\text{slope } \overrightarrow{BC} = \frac{6-0}{3-5} = -3$$

$$\text{slope } \overrightarrow{AC} = \frac{6-1}{3-2} = 5$$

None are negative reciprocals of each other.

Not a right \triangle .

9. (10 points) Find the equation of a line passing through $(4, 5)$ and perpendicular to the line $3x + 5y = 8$. Put your answer in slope-intercept form.

$$5y = -3x + 8$$

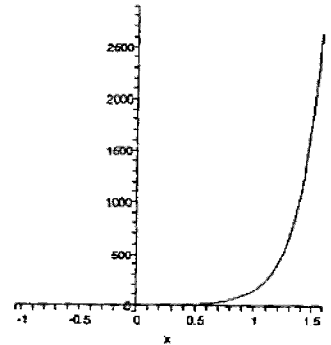
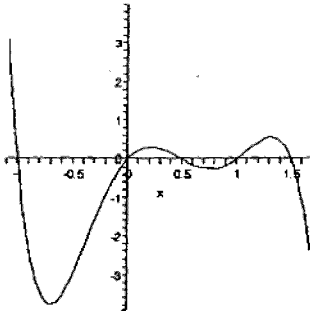
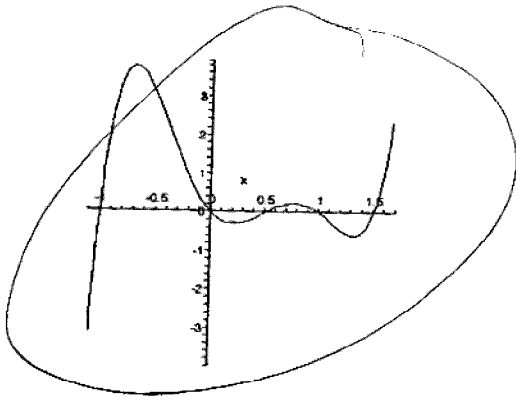
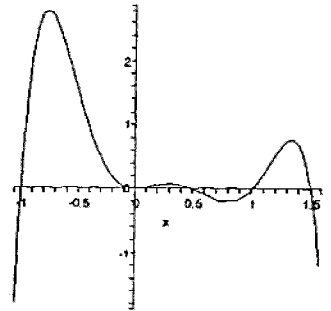
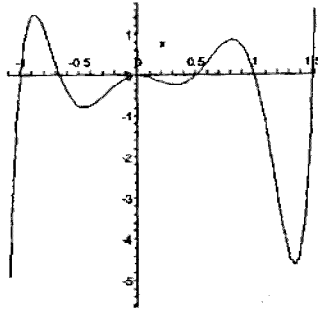
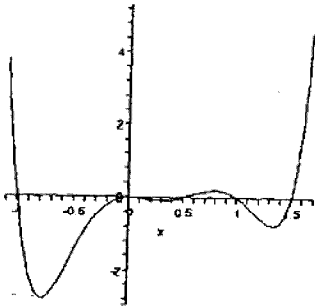
$$y = -\frac{3}{5}x + \frac{8}{5} \quad \text{so our line has slope } \frac{5}{3}$$

$$y - 5 = \frac{5}{3}(x - 4)$$

$$y - 5 = \frac{5}{3}x - \frac{20}{3}$$

$$y = \frac{5}{3}x - \frac{5}{3}$$

10. (6 points) Circle the graph of $y = 4x^5 - 8x^4 - x^3 + 8x^2 - 3x$.



11. (10 points) Neatly sketch the graph of $f(x) = x^2 + 2x - 15$. Label the vertex, axis and all x and y intercepts. Determine the domain and the range of $f(x)$.

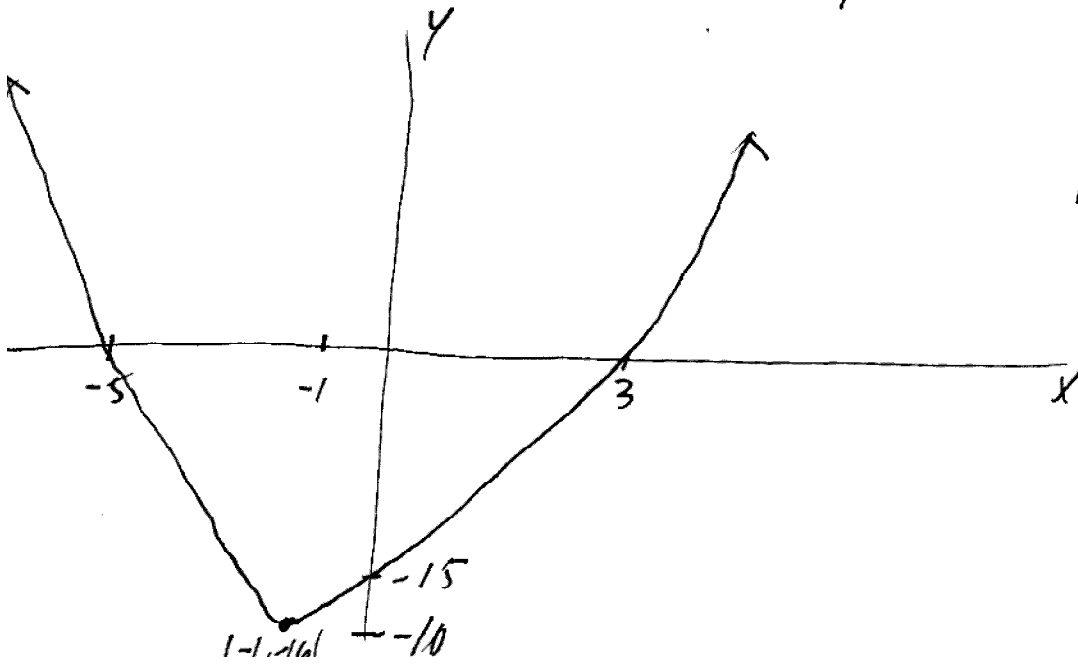
vertex $x = \frac{-2}{2} = -1$ axis $x =$

$y = (-1)^2 + 2(-1) - 15 = -16$

roots: $(x-3)(x+5) = 0$

$x = 3, x = -5$

axis y -int = $f(0) = -15$ $(0, -15)$



Domain = $(-\infty, \infty)$

Range = $[-16, \infty)$