

Recall

Product Rule: $\frac{d}{dx}(f(x)g(x)) = \frac{d}{dx}f(x) \cdot g(x) + f(x) \cdot \frac{d}{dx}g(x)$

i.e. $(fg)' = f'g + fg'$

Example

Find tangent line to $y = \sqrt{x} \sin x$ at $x = \pi$

Example

$g(t) = (t^3 + 1) \cos t \sqrt{t}$ Find $g'(t)$

Quotient Rule

Let $F(x) = \frac{f(x)}{g(x)}$ If $f(x)$ & $g(x)$ are differentiable

then so is $F(x)$ and

$$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

"lo de hi minus hi de lo over lo squared"

Pract Similar add & subtract tricks

Examples

$$f(x) = \frac{1}{x} \quad f'(x) = \frac{x \cdot 0 - 1 \cdot 1}{x^2} = -\frac{1}{x^2} \quad \text{agrees w/ power rule}$$

$$f(x) = \frac{2x+1}{x-5} \quad f'(x) = \frac{(x-5)(2) - (2x+1)(1)}{(x-5)^2} = \frac{-11}{(x-5)^2}$$

$$g(t) = \frac{t^2}{1-t} \quad g'(t) = \frac{(1-t)2t - t^2(-1)}{(1-t)^2} = \frac{-t^2 + 2t}{(1-t)^2}$$

$$f(x) = \frac{\sqrt{x} \sin x}{x^2 - \cos x} \quad f'(x) = \frac{(x^2 - \cos x) \left(\frac{1}{2} x^{-1/2} \sin x + \sqrt{x} \cos x \right) - \sqrt{x} \sin x (2x - \sin x)}{(x^2 - \cos x)^2}$$

Find tangent line to $y = \frac{x^2-1}{x+5}$ at $x=2$

Derivatives of Trig Functions

$$f(x) = \sin x \quad f'(x) = \cos x$$

$$f(x) = \cos x \quad f'(x) = -\sin x$$

$$f(x) = \tan x = \frac{\sin x}{\cos x} \quad f'(x) = \frac{\cos x \cos x - \sin x (-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$f(x) = \cot x = \frac{\cos x}{\sin x} \quad f'(x) = -\csc^2 x = -\sec^2 x$$

$$f(x) = \sec x = \frac{1}{\cos x} \quad f'(x) = \frac{\cos x \cdot 0 - 1(-\sin x)}{\cos^2 x} = +\tan x \sec x$$

$$f(x) = \csc x \quad f'(x) = -\csc x \cot x$$

Chain Rule

We still can't differentiate most functions e.g.

$$f(x) = \sin(x^2)$$

$$h(t) = \sqrt{t^2 + 3t}$$

$$g(s) = (\cos s)^3$$

Recall Suppose $g(x)$, $f(x)$ given. The composition is

$$f \circ g = f(g(x)).$$

Theorem (Chain Rule) Let

$$F(x) = f(g(x)). \text{ Then}$$

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Example $y = \sin(x^2)$ $f(x) = \sin x$ $g(x) = x^2$
 $f'(x) = \cos x$ $g'(x) = 2x$

$$\begin{aligned} \frac{dy}{dx} &= f'(g(x)) \cdot g'(x) \\ &= \cos(x^2) \cdot 2x \end{aligned}$$

Example $h(t) = (t^2 + 1)^{10}$

outside in!