P. 119 — # 3, 4, 7, 10, 14, 16, 23, 26, 29, 34, 40, 44, 48, 49, 50 (only even numbers)

10/5/06

Differentiation Rules Review

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}(x^n) = nx^{n-1}$ any $n \in \mathbb{R}$

3. $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$, i.e. $(cf(x))' = cf'(x)$

4. $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$ i.e. $(f \pm g)' = f' \pm g'$

5. Product Rule $\frac{d}{dx}(f(x)g(x)) = \frac{d}{dx}f(x)\,g(x) + f(x)\frac{d}{dx}g(x)$

$$(fg)' = f'g + fg'$$

6. Quotient Rule $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$

i.e. $$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

7. Chain Rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

Chain Rule

$f(g(x))$ is "f of something" ; its derivative is

$f'($something$)$ · derivative of something

Examples

$g(t) = \sqrt{\frac{\cos t}{t^2+1}}$ $\quad g'(t) = \frac{1}{2\sqrt{\frac{\cos t}{t^2+1}}} \cdot \frac{(t^2+1)(-\sin t) - \cos t \cdot 2t}{(t^2+1)^2}$

$f(x) = (x^2+1)^{40}$ $\quad f'(x) = 40(x^2+1)^{39} \cdot 2x$

$h(x) = \sqrt{\cos(x^2)}$

$h'(x) = \frac{1}{2\sqrt{\cos(x^2)}} \cdot (-\sin(x^2)) \cdot 2x$

Notice $h(x) = f(g(u(x)))$ $\quad f(x) = \sqrt{x}$, $g(x) = \cos x$, $u(x) = x^2$

$h'(x) = f'(g(u(x))) \cdot g'(u(x)) \cdot u'(x)$

Alternate Form

Suppose $g(x) = u$ $\quad y = f(u) = f(g(x))$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{dy}{dx}$$

$$f'(u) \quad g'(x) = f'(g(x)) \, g'(x)$$

Example

$y = u^2 + 2u$

$u = \sin(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{dy}{dx} = (2u + 2)\cos x$$

$$= (2\sin x + 2)\cos x$$

Alternately: $y = \sin^2 x + 2\sin x$

$$\frac{dy}{dx} = 2\sin x \cos x + 2\cos x$$

Warning

$\sin^2(x) = (\sin x)^2$, it is (something)^2

Problem

$y = \sin(\sin(x))$ $\quad y' = \cos(\sin(x)) \cdot \cos x$

$y = (x\cos x)^{10}$ $\quad y' = 10(x\cos x)^{?}(\cos x - x\sin x)$

Ans

$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

Problem Prove derivative of even is odd, vice versa.

$y = \tan\left(\frac{x}{1+x}\right)$

$y = \csc(2x)$