Exam #2
Friday October 20
Chapt. 2.1-2.7

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10/19/06

- go over quiz, must learn differ. rules and interpretation of derivative.

<u>Problem</u> Find tangent line to $y = x\sin x$ at $x = \pi/4$.

Notice $y = f(x)$. Thus point is $(\pi/4, f(\pi/4)) = (\pi/4, \frac{\pi}{4}\cdot\frac{\sqrt{2}}{2})$
$= (\pi/4, \frac{\pi}{8}\sqrt{2})$

Slope is $f'(\pi/4)$!

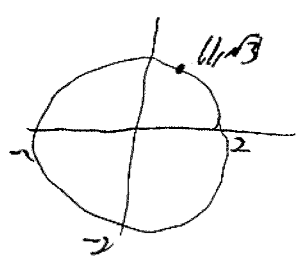
$f'(x) = \sin x + x\cos x$

$f'(\pi/4) = \frac{\sqrt{2}}{2} + \frac{\pi}{4}\frac{\sqrt{2}}{2} = \sqrt{2}(1/2 + \pi/8)$

Answer $y - \frac{\pi}{8}\sqrt{2} = \sqrt{2}(\frac{1}{2} + \pi/8)(x - \pi/4)$

Thus when $y = f(x)$ it is easy to find eq. of tangent line.

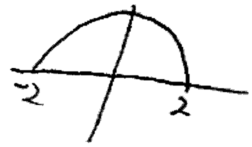
<u>Problem</u>
Find eq of tangent line to $x^2 + y^2 = 4$ at point $(1, \sqrt{3})$!



- Not of the form $y = f(x)$
- still seems it should have a tangent line.

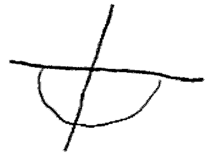
In this example we can solve for y:

$y = \sqrt{4-x^2}$



top half

$y = -\sqrt{4-x^2}$



bottom half

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \cdot -2x = \frac{-x}{\sqrt{4-x^2}}$$

plug in $x=1$ slope $= \frac{-1}{\sqrt{3}}$

$$\boxed{y - \sqrt{3} = \frac{-1}{\sqrt{3}}(x-1)}$$

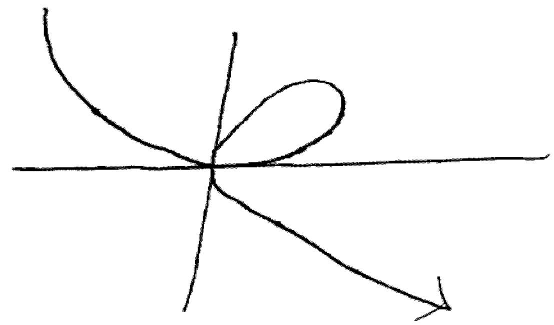
Suppose we want $(+1, -\sqrt{3})$, now use

$y = -\sqrt{4-x^2}$ $y' = \frac{x}{\sqrt{4-x^2}}$ slope $= \frac{1}{\sqrt{3}}$

Thus $x^2+y^2=25$ can be handled by:

1. Write y as a function of x
2. Proceed as before

<u>Harder example</u> $x^3+y^3=6xy$



- Hard to solve for y
- Still near almost any point, y is <u>implicitly</u> a function of x.
- should be a tangent line.

<u>Implicit Differentiation</u>

Given: An equation relating x and y but not of the form $y=f(x)$.

<u>Procedure</u>: Assume y is a function of x and differentiate both sides with respect to x. Solve for y' if possible (perhaps after plugging in).

<u>Warning</u>: $\frac{d}{dx}x = 1$

$\frac{d}{dx}y = \frac{dy}{dx}$ a.k.a. y' since

we assume $y=y(x)$.

Example

Find eq. of tangent line to $x^3+y^3=6xy$ at $(3,3)$.

- Notice: given x value, may be no y's or many y's on the graph.

$x^3+y^3=6xy$ Think: $x^3+y(x)^3=6xy(x)$

Differentiate

$$3x^2+3y^2\cdot\frac{dy}{dx}=6y+6x\frac{dy}{dx}$$

$$\frac{dy}{dx}(6x-3y^2)=3x^2-6y$$

$$\frac{dy}{dx}=\frac{3x^2-6y}{6x-3y^2}=\frac{x^2-2y}{2x-y^2}$$

- Slope at $(3,3)$ is $\frac{3}{-3}=-1$

$$\boxed{y-3=-(x-3)}$$

Problem: Horizontal tangents?

$x^2-2y=0$ $y=\frac{x^2}{2}$

$x^3+\frac{x^6}{8}=3x^3$

$16x^3=x^6$

$16=x^3$ $x=\sqrt[3]{16}$ $y=$ ___

Problem: Vertical tangents?