p. 125 9, 10, 13, 22, 24, 36, 40a

10/10/06

Review Given: an equation involving y and x
Procedure: Differentiate with respect to x.
Assume: $y = y(x)$, i.e. y is a function of x. Thus

$$\frac{d}{dx}(x) = 1 \quad \text{as usual}$$

$$\frac{d}{dx}(y) = y' \quad \text{a.k.a.} \quad \frac{dy}{dx}$$

Example

$$x^2y + xy^2 = 3x \qquad \text{Find } y' \text{ and } y''$$

Solution

$$2xy + x^2y' + y^2 + 2xyy' = 3$$

$$y'(x^2+2xy) = 3 - 2xy - y^2$$

$$y' = \frac{3-2xy-y^2}{x^2+2xy}$$

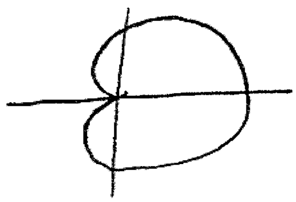
$$y'' = \frac{(x^2+2xy)(-2xy' - 2y - 2yy') - (3-2xy-y^2)(2x+2y+2xy')}{(x^2+2xy)^2}$$

Now can sub in for $y' = \dfrac{3-2xy-y^2}{x^2+2xy}$

to get y'' in terms of x & y.

<u>Problem</u>

Find tangent line to $x^2+y^2=(2x^2+2y^2-x)^2$ at $(0,\frac{1}{2})$



<u>Solution</u> $2x+2yy'=2(2x^2+2y^2-x)(4x+4yy'-1)$

<u>Observe</u> we could solve for y', but why bother, we only need y' when $x=0$ $y=1/2$

$$0+y'=2\left(\tfrac{1}{2}\right)(2y'-1)$$

$$y'=2y'-1$$

$$y'=1$$ Thus <u>at the point $(0,\frac{1}{2})$</u> $y'=1$

$$y-\tfrac{1}{2}=1(x-0)$$

$$y=x+\tfrac{1}{2}$$

<u>Rmk</u> Suppose we wanted horizontal tangents then we would solve for y' and set $=0$.

<u>Problem</u> Tangent and normal lines to $y^2=5x^4-x^2$ at $(1,2)$

$$2yy'=20x^3-2x \qquad y'=\frac{20x^3-2x}{2y}=\frac{10x^3-x}{y}$$

So y' at $(1,2)$ is $9/2$

Tangent line $\quad y-2=9/2\,(x-1)$

Normal line $\quad y-2=-2/9\,(x-1)$

#41 Find all points on $x^2y^2+xy=2$ where slope of tangent is -1

$$2xy^2+2x^2yy'+y+xy'=0$$

$$y'=\frac{-2xy^2-y}{2x^2y+x}$$

$$-1=\frac{-2xy^2-y}{2x^2y+x}$$

$$-2x^2y-x=-2xy^2-y$$

$$2xy^2-2x^2y=x-y$$

$$2xy(y-x)=x-y$$

$$2xy=-1 \quad \text{or} \quad x=y$$

$2xy=-1$

$y=-\frac{1}{2x}$

$x^2\left(\frac{1}{4x^2}\right)+-\frac{1}{2}=2$ NO SOLUTION

$x=y$

$x^4+x^2=2$

$x^4+x^2-2=0$

$$x^2=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm3}{2}=-2,\ 1$$

$x=\pm1$

$\boxed{(1,1) \quad (-1,-1)}$

Problem

32 Show $x^2+y^2=r^2$, $ax+by=0$ are orthogonal

35 $y=cx^2$ $\quad x^2+2y^2=K$

33 $2x+2yy'=0$

$y'=-\frac{x}{y}$

$ax+by=0$

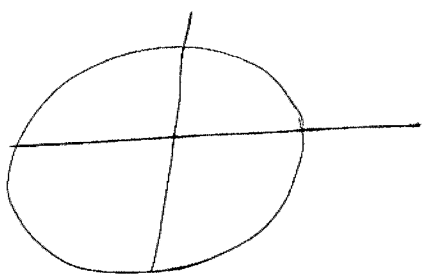
$a+by'=0$

$y'=-\frac{a}{b}$

$x=-a$ then

$y=a^2/b$

so $-\frac{x}{y}=\boxed{\frac{b}{a}}$



35. $y'=2cx$

$2x+4yy'=0$

$y'=\frac{-2x}{4y}=-\frac{x}{2y}$

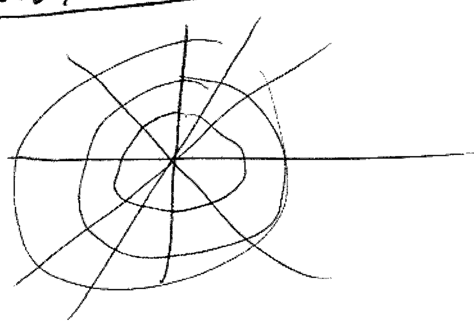
at a point (x, cx^2)

$y'=2cx$

$y'=\frac{-x}{2cx^2}=\frac{-1}{2cx}$

Sketches

33



35.