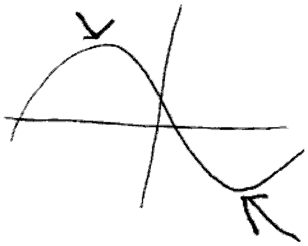
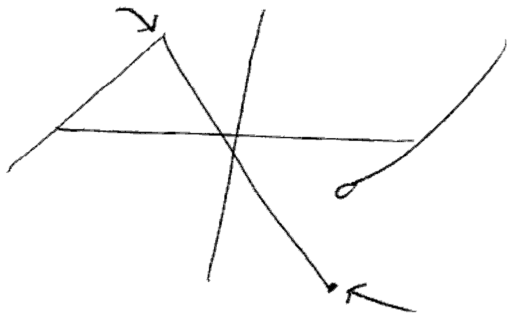
HW p. 147 3-4, 11, 12, 14

10/23/06

Observation It appears that a function has it's maximum or minimum values at places where either $f'(x)=0$ or f' Does not exist





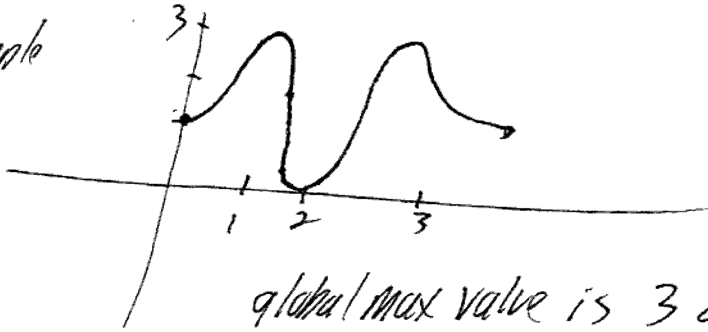
Plan

1. Make precise what we mean by maximum and minimum values
2. Determine when functions have these values.
3. Tie to calculus by proving the observation above
4. Solve all kinds of optimization problems

Def Let $f(x)$ have domain $D \subseteq \mathbb{R}$. f has an absolute (aka global) maximum at c if $f(c) \geq f(x)$ for all $x \in D$. $f(c)$ is the maximum value.

Similarly absolute minimum, minimum value, extreme value means global max or min

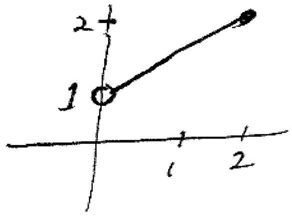
Example



global max value is 3 at $x=1$ & $x=3$

global min value is 0 at $x=2$

Example

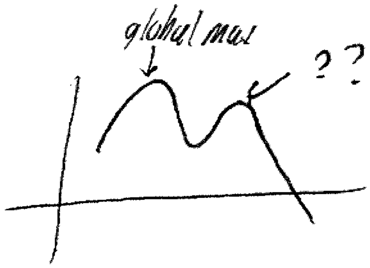


$D = (0, 2]$

max value is 2 at $x = 2$

no minimum value!

Example



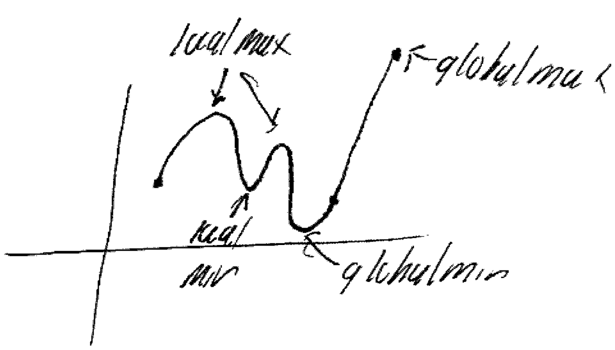
Def $f(x)$ has a local maximum at $x = c$ if

$f(c) \geq f(x)$ for all x "near c"

i.e. $\exists\, \varepsilon$ so $f(c) \geq f(x)$ for all x in $(c-\varepsilon, c+\varepsilon)$!

Similarly local minimum if $f(c) \leq f(x)$ for all x near c

Example

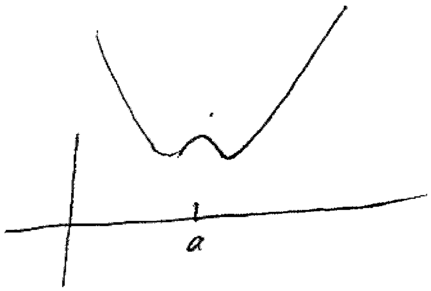


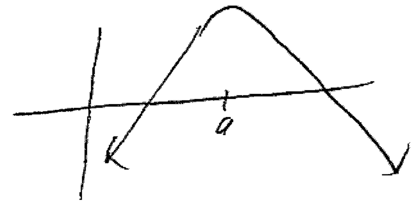
Example

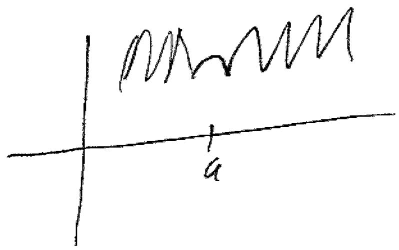
Key Remark Calculus will only find local max/mins, won't tell us global info.

$f'(a)$ is completely determined by points near a.

Example







All have same $f'(a)$!

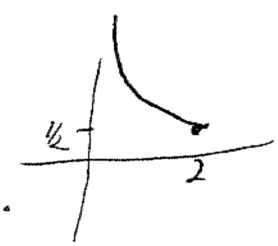
Step 2

Theorem Let f be continuous on a closed interval $[a,b]$.
Then f attains ~~its~~ a maximum and minimum value on $[a,b]$.

Rmks
1. Proof is hard!
2. 2^{nd} thm about continuous functions on closed interval!

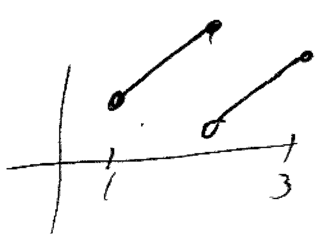
Examples

1. $f(x) = \frac{1}{x}$ on $(0,2]$



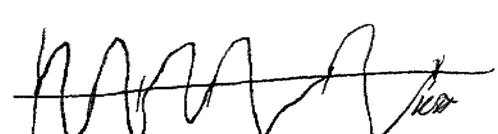
has minimum value $1/2$
No maximum! $(0,2]$ not a closed interval!

2.



No minimum value
not continuous.

3. $y = \sin x \quad [0, 100\pi]$

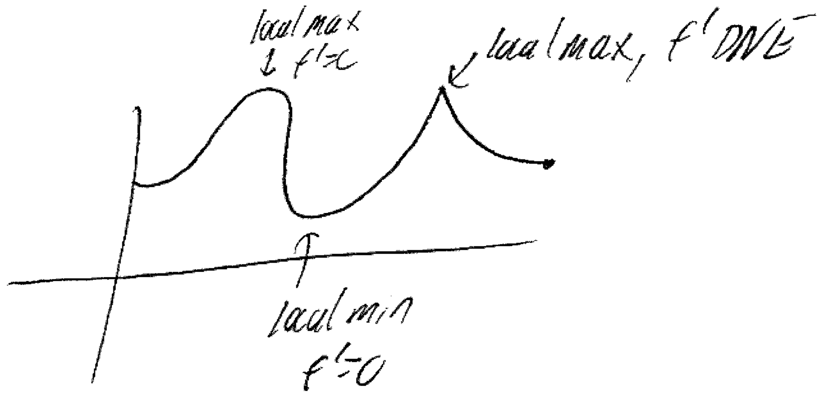


Attains max value 1, min value -1 many times

Theorem

Suppose f has a local maximum ~~and~~ or a local minimum at $x=c$.
Then if $f'(c)$ exists it must $=0$.

Example



Sketch of proof.