p. 147 # 18, 22, 28, 30, 36, 40, 41
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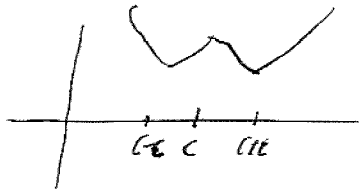
10/24/06

Review Given $f(x)$ with domain D. $f(x)$ has a global maximum at c if $f(c) \geq f(x)$ for all $x \in D$. $f(c)$ is called the maximum value.
Same for global minimum.

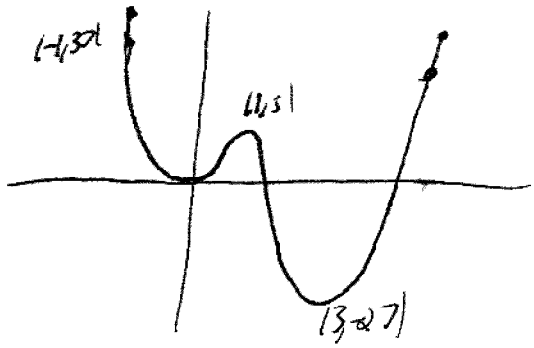
Def max and min values are called extreme values.

Def $f(x)$ has a local maximum if $f(c) \geq f(x)$ $\forall x$ in $(c-\varepsilon, c+\varepsilon)$, $\varepsilon > 0$,



local minimum similarly. Rmk local ⇒ local

Example $y = 3x^4 - 16x^3 + 18x^2$ on $[-1, 4]$

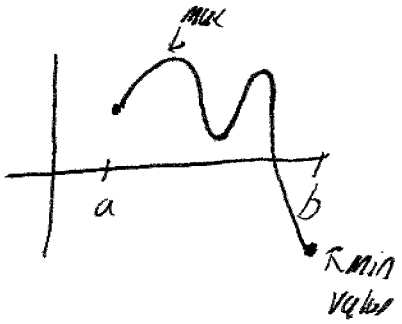


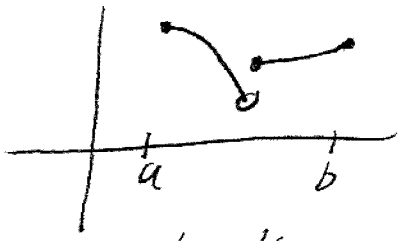
- Global max value of 37 at $x = -1$
- local min value of 0 at $x = 0$
- local max at $x = 1$
- global min at $x = 3$

Theorem Suppose $f(x)$ is continuous on a closed interval $[a, b]$. Then f attains a maximum value $f(c)$ and a minimum value $f(d)$ for some $c, d \in [a, b]$.

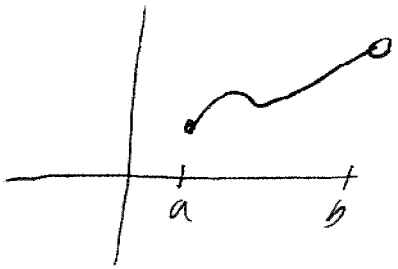
Rmk This is the 2nd Thm about continuous functions on a closed interval.

Examples





Not continuous, no minimum



continuous but domain not a closed interval, no maximum

Rmk This is an existence theorem, doesn't tell how to find c, d

Fermat's Thm

Suppose f has a local max or local min at c.
If $f'(c)$ exists then $f'(c) = 0$

Example Find absolute max and min.

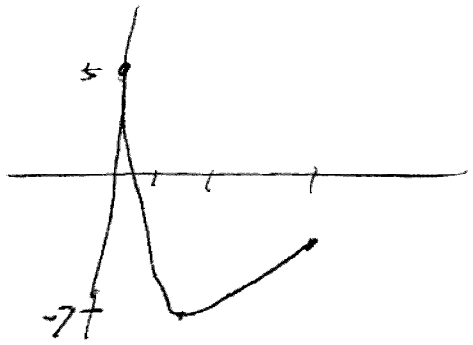
$f(x) = 3x^2 - 12x + 5$ on $[0, 3]$.

$f'(x) = 6x - 12$ so $6x - 12 = 0$ $\quad x = 2$

No points where $f'(x)$ DNE.

x	$f(x)$
0	5
2	-7
3	-4

global max is 5
global min -7



Example

Show that $c = 2$ is a critical value of $f(x) = 3 + (x-2)^3$ but $f(x)$ does not have a local max or min.

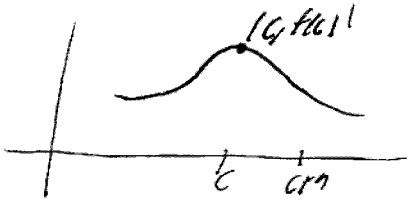
Example

Find critical numbers for $h(t) = \frac{t+1}{t^2+t+1}$

Rmk must be in the domain

Proof Suppose $f(c)$ is a local max and

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \text{ exists.}$$



When $h > 0$ then $\frac{f(c+h)-f(c)}{h}$ is negative

When $h < 0$ then " a positive. So limit must be 0,

Finding Absolute max & min.

Procedure: Let $f(x)$ be continuous on $[a,b]$. To find max and min values:

1. Find c's in (a,b) where $f'(c) = 0$ or $f'(a)$ DNE (critical values)
   plug in these c's

2. Plug in $f(a)$, $f(b)$.

3. Find max & min.