p. 154 6, 7, 11, 14, 15, 18

Recall

- If $f(x)$ has a local max or min at c then $f'(c)=0$ or $f'(c)$ DNE
- Continuous functions on a closed interval have global extrema.

Thus to find these extreme values we find critical #'s and test them together with the endpoints

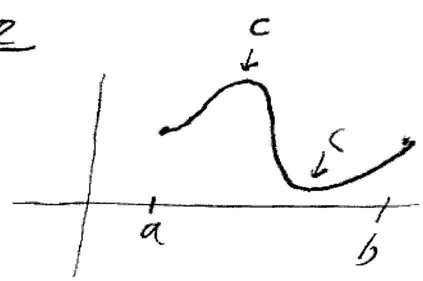
A little theory...

Rolle's Theorem Suppose:

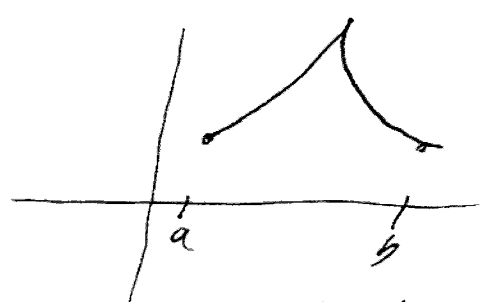
1. f is continuous on $[a,b]$.
2. f is differentiable on (a,b)
3. $f(a)=f(b)$

Then there exists $c \in (a,b)$ with $f'(c)=0$.

Picture



may be many c's
only one is guaranteed



Rolle's Thm does not apply, f is not differentiable on all of (a,b).

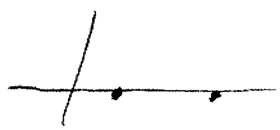
<u>Proof</u> If $f(x)=k$ then $f'=0$ so any c works.
Thus assume $f(x)$ is not constant.
Let $f(x)>f(a)$ for some $x\in(a,b)$. Then max value occurs at $c\in(a,b)$ so $f'(c)=0$. Similar for $f(x)<f(a)$

<u>Example</u> Prove $x^3+x-1=0$ has exactly one real root.

$f(0)=-1$

$f(1)=1$ Thus by IVT there is a root between 0 and 1.

If two roots then $f'=0$ some pt.



two roots $\Rightarrow f'(c)=0$ some c

But $f'(x)=3x^2+1>0\ \forall x$.

<u>Mean Value Theorem</u> Suppose $f(x)$ is continuous on $[a,b]$, differentiable on (a,b). Then there exists some $c\in(a,b)$ with

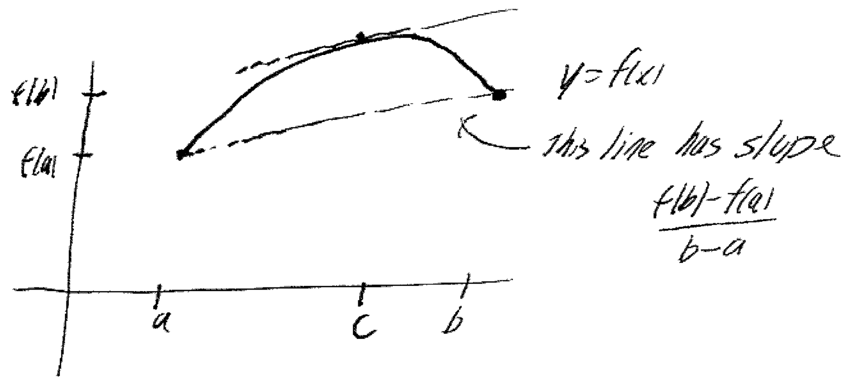
$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

<u>Comments</u>

1. This is saying at some c the instantaneous ROC = avg rate of change.

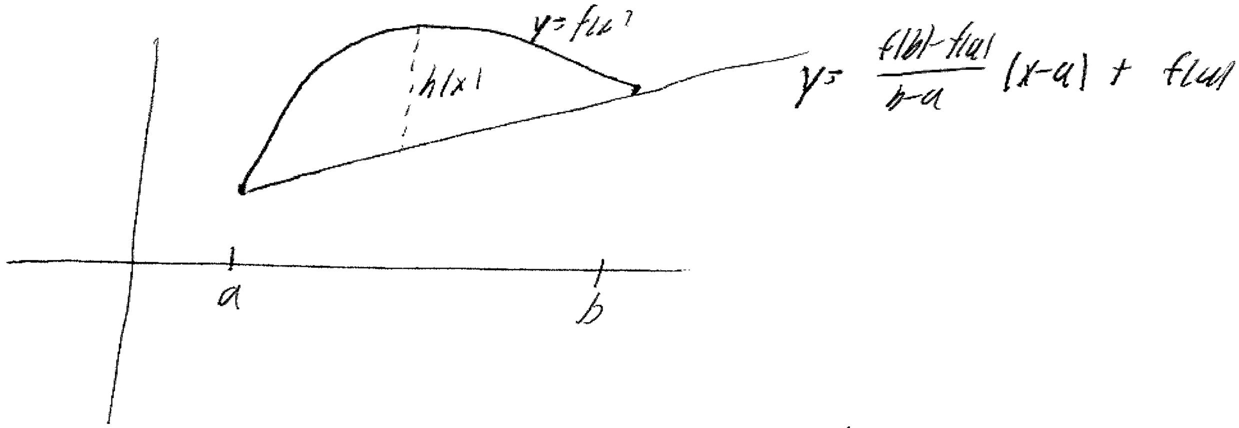
2. MVT $\Rightarrow$ Rolles

MVT Illustration



MVT is an existence theorem.

Proof



Let $h(x)$ be distance from $f(x)$ to secant line.

$$h(x) = f(x) - f(a) - \frac{f(b)-f(a)}{b-a}(x-a)$$

- $h(a) = h(b) = 0$

Apply Rolles to $h(x)$ so $h'(c) = 0$ some c.

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

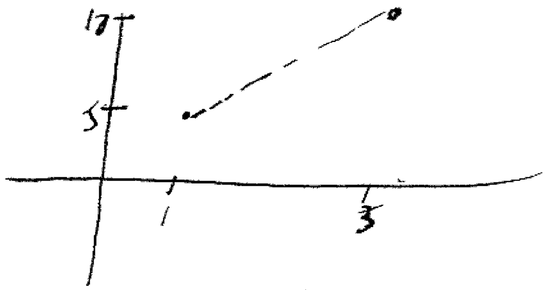
$$0 = f'(c) - \frac{f(b)-f(a)}{b-a}$$

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

Example

$f(x) = x^3 - 2x^2 + x + 5$ on $[1,3]$

$f(1) = 5 \quad f(3) = 18 \qquad \frac{f(1) - f(3)}{1-3} = \frac{-13}{-2} = 13/2$



This MVT guarantees at least one $c \in (1,3)$ w/ $f'(c) = 13/2$

$f'(x) = 3x^2 - 4x + 1$

$13/2 = 3x^2 - 4x + 1$

$3x^2 - 4x - 11/2 = 0$

$$x = \frac{4 \pm \sqrt{16 + 66}}{6} = \frac{4 \pm \sqrt{82}}{6} \approx \frac{13.1}{6}, \frac{-5.1}{6}$$

c!!

COROLLARY

Suppose $f'(x) = 0 \ \forall x \in (a,b)$. Then f is constant.

COROLLARY

Suppose $f'(x) = g'(x) \ \forall x \in (a,b)$. Then $f(x) = g(x) + c$ for some c.