HW Read 3.3, 3.4
p.161 2, 4, 8, 10, 12, 13

10/30/06

Recall $f'(a)$ is the slope of the tangent line to the graph $y=f(x)$ at the point $(a, f(a))$.

Question What information about the graph can we get from $f'(x)$.

So far: local max/mins occur where $f'(x)=0$ or DNE.

Recall $f(x)$ is increasing on an interval I if
$a<b$, $a,b\in I \Rightarrow f(a)<f(b)$.
$f(x)$ is decreasing on I if $a<b$, $a,b\in I \Rightarrow f(a)>f(b)$.

Theorem
a. If $f'(x)>0$ on an interval then $f(x)$ is increasing on the interval
b. If $f'(x)<0$ on an interval then $f(x)$ is decreasing on the interval

Proof a. Let $x_1<x_2$ in the interval. By MVT

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = f'(c)(x_2-x_1) \quad \text{some } c$$

but $f'(c)>0$ so $f(x_2)-f(x_1)>0$.
b. Reverse

Example Find intervals on which $f(x)=2x^3-3x^2-36x+5$ is increasing and decreasing.

$f'(x) = 6x^2 - 6x - 36$

Set $0 = 6x^2 - 6x - 36$

$0 = x^2 - x - 6 = (x-3)(x+2)$

$$\overset{+}{\longleftarrow\!\!-\!\!-}\underset{-2}{|}\overset{-}{-\!\!-\!\!-}\underset{3}{|}\overset{+}{-\!\!-\!\!\longrightarrow}$$

$f(x)$ is increasing on $(-\infty, -2) \cup (3, \infty)$
decreasing on $(-2, 3)$

Question: Suppose $f'(c) = 0$. Does $f(x)$ have a local min/max at c? What if $f'(c)$ DNE?

First Derivative Test: Let $f(x)$ be continuous and let c be a critical # for $f(x)$.

a. If f' changes from pos. to negative at c then $f(x)$ has a local max at c.

b. If f' changes from neg to positive at c then $f(x)$ has a local min at c.

c. If f' does not change sign, $f(x)$ has neither a local max or min at $x = c$.

Example: $f(x) = 2x^3 - 3x^2 - 36x + 5$ has a local max at $x = -2$, local min at $x = 3$

Example: $f(x) = x^3 - 6x^2 + 12x - 7$

$f'(x) = 3x^2 - 12x + 12 = 3(x-2)^2$

$$\overset{+}{\longleftarrow\!\!-\!\!-}\underset{2}{|}\overset{+}{-\!\!-\!\!\longrightarrow}$$

$f(x)$ has neither local max nor local min at $x = 2$

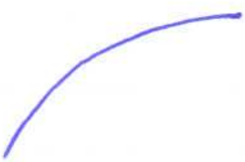
Example $f(x) = x + \sqrt{1-x}$. Find local max/mins using first der. test.

Concavity

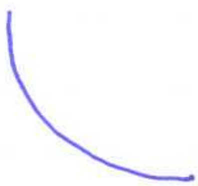
Q What does the second derivative $f''(x)$ tell us?

increasing and concave up

$f'(x)$ is >0 and $f'(x)$ is increasing. Thus $f''(x) > 0$.



$f'(x) > 0$
$f''(x) < 0$
increasing and concave down

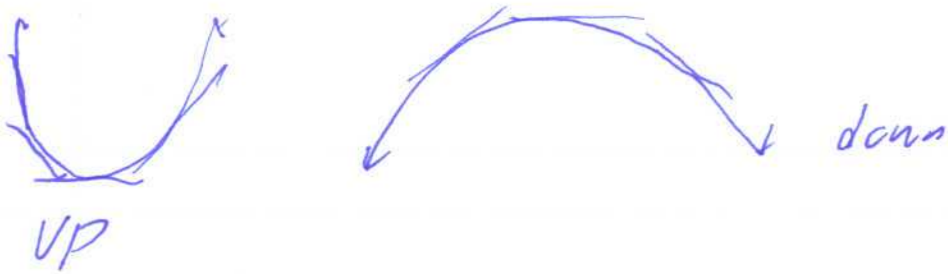


$f'(x) < 0$
$f''(x) > 0$
decreasing and concave up

$f'(x) < 0$
$f''(x) < 0$
decreasing and concave down

Formal Definition $f(x)$ is concave upward on I if graph lies above its tangent line at all points on I.

$f(x)$ is concave downward on I if graph lies below its tangent line on I.



Def. A point P on $y=f(x)$ is an inflection point if $f(x)$ is continuous and curve changes concavity.

Ex.

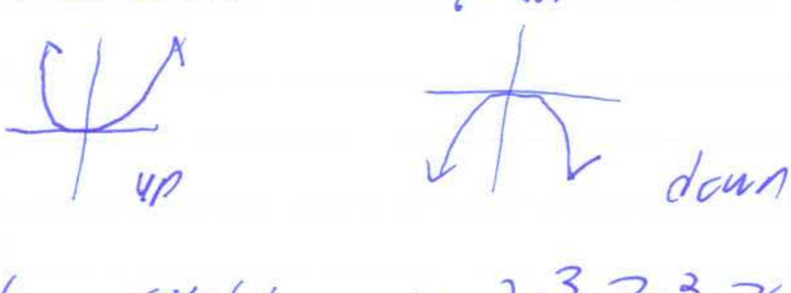
Thm (a) If $f''(x) > 0$ on I then graph is concave up on I

(b) If $f''(x) < 0$ on I then graph is concave down on I

Easy to remember!

$y = x^2$ $\qquad$ $y = -x^2$

$f''(x) = 2 > 0$ $\qquad$ $f''(x) = -2$



Problem Sketch $y = 2x^3 - 3x^2 - 36x + 5$.