10/31/06

Review: Suppose $f(x)$ is continuous and you want to know intervals where $f(x)>0$ or $f(x)<0$.

- Set $f(x)=0$, divide up # line
- test one point in each interval, by IVT you are done.

Theorem Let I be an interval

a. If $f'(x)>0$ on I then $f(x)$ is increasing on I

b. If $f'(x)<0$ on I then $f(x)$ is decreasing on I

c. If $f''(x)>0$ on I then $f(x)$ is concave up on I

d. If $f''(x)<0$ on I then $f(x)$ is concave down on I

Example 6 $f(x)=x^{2/3}(6-x)^{1/3}$ roots $x=0$ $x=6$

$$f'(x)=\tfrac{2}{3}x^{-1/3}(6-x)^{1/3}+x^{2/3}\cdot\tfrac{1}{3}(6-x)^{-2/3}\cdot -1$$

$$=\frac{2(6-x)^{1/3}}{3\sqrt[3]{x}}-\frac{x^{2/3}}{3(6-x)^{2/3}}$$ DNE at $x=0, 6$

Now set $f'(x)=0$

$$0=\frac{2(6-x)^{1/3}}{3\sqrt[3]{x}}-\frac{x^{2/3}}{3(6-x)^{2/3}}$$

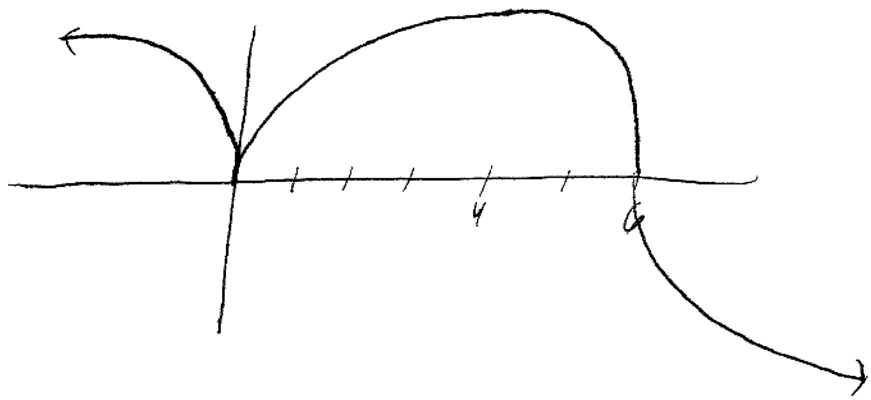
$$0=2(6-x)-x$$
$$=12-3x \qquad x=4$$

$\boxed{\text{critical \#s } 0, 4, 6}$

$$f'(x)=\frac{4-x}{\sqrt[3]{x}\,(6-x)^{2/3}} \qquad f''(x)=\frac{-8}{x^{4/3}(6-x)^{5/3}}$$

	$f'(x)$	$f''(x)$
$x<0$	$-$	$-$
$0<x<4$	$+$	$-$
$4<x<6$	$-$	$-$
$6<\infty$	$-$	$+$

vertical tangent at $x=0, x=6$



Def. An <u>inflection point</u> on graph of $y=f(x)$ is a place where the curve changes from concave up to concave down or vice versa.

<u>Example</u> 6, 0 on curve above.

<u>Rmk</u> At an inflection point we must have $f''(c)=0$ or $f''(c)$ DNE.

<u>SECOND DERIVATIVE TEST</u>

Suppose $f''(x)$ is continuous near $x=c$.

a. If $f'(c)=0$ and $f''(c)>0$ then $f(x)$ has a local minimum at $x=c$.

b. If $f'(c)=0$ and $f''(c)<0$ then $f(x)$ has a local maximum at $x=c$.

** If $f''(c)=0$ also then anything is possible! e.g. $y=x^3, x^4, -x^4$ at $x=0$

Example $f(x) = x^4 - 6x^2$ $\qquad g(x) = \frac{x^2}{x^2-1}$

$f'(x) = 4x^3 - 12x = 4x(x^2-3)$

critical #'s $0, \sqrt{3}, -\sqrt{3}$

$\underset{}{\overset{-}{\longleftarrow}} -\sqrt{3} \overset{+}{} \; 0 \overset{-}{} \; \sqrt{3} \overset{+}{\longrightarrow}$

decreasing $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$
increasing $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

local min at $-\sqrt{3}$ by 1st der test, also $\sqrt{3}$
local max at 0 by 1st der test.

$f''(x) = 12x^2 - 12 = 12(x+1)(x-1)$

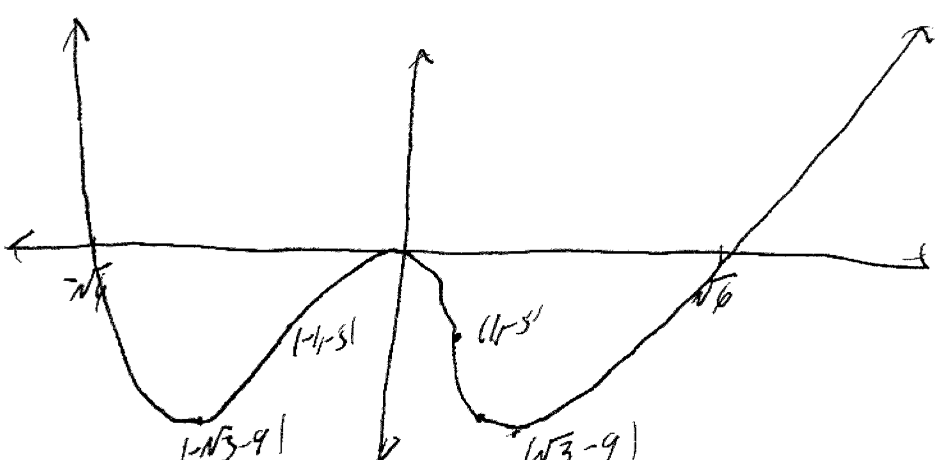
$\overset{+}{\longleftarrow} -1 \overset{-}{} \; 1 \overset{+}{\longrightarrow}$

concave up $(-\infty, -1) \cup (1, \infty)$
concave down $(-1, 1)$

inflection points $(-1, -5)$
$(1, -5)$

Notice $f''(\sqrt{3}) > 0$ so local min by 2nd der. test
$f''(0) < 0$ so local max by 2nd " "
$f''(-\sqrt{3}) > 0$ so local min

Roots at $0, \pm\sqrt{6}$



Exam

Summary

1st Derivative

- places where $f'(c)=0$ or $f'(c)$ DNE called critical #'s
- $f'(x)>0$ on $I \Rightarrow$ increasing
- $f'(x)<0$ on $I \Rightarrow$ decreasing
- local max/mins occur at critical #'s
- 1st derivative test classifies if critical # gives max, min or neither.

2nd Derivative

- $f''(x)>0$ on I means concave up on I
- $f''(x)<0$ on I means concave down on I
- Inflection point where concavity switches
  must have $f''(c)=0$ or $f''(c)$ DNE
- 2nd derivative test sometimes works
  - doesn't work if $f'(c)$ DNE
  - gives no info if $f''(c)=0$

- Example given graph of $f'(x)$
- $f(x)=\dfrac{x^2}{x^2+4}$