11/2/07

Review

First Derivative

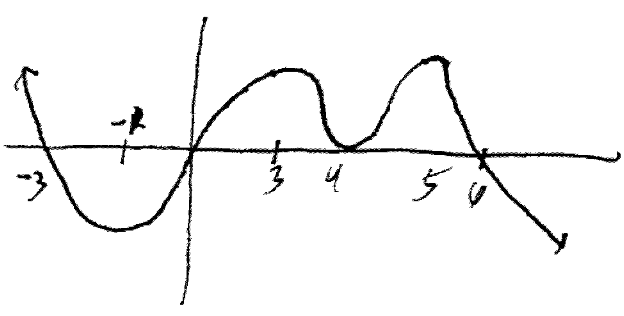
- critical # means $f'(c)=0$ or DNE
- first derivative test classifies whether $f(x)$ has local min/max at critical #
- $f'(x)>0$ on I means $f(x)$ is increasing on I
- $f'(x)<0$ on I means $f(x)$ is decreasing on I
- $f'(x)=0$ on I implies $f(x)$ is constant on I

Second derivative

- $f''(x)>0$ on I means $f(x)$ is concave up on I.
- $f''(x)<0$ on I means $f(x)$ is concave down on I
- inflection points where concavity changes
  - must have $f''=0$ or DNE.
- $f''(x)=0$ on I means $f'(x)$ constant $\Rightarrow f(x)=ax+b$
- 2^{nd} derivative test can classify some places where $f'(c)=0$ as local max or min, but if $f''(c)=0$ also, no information.

Example

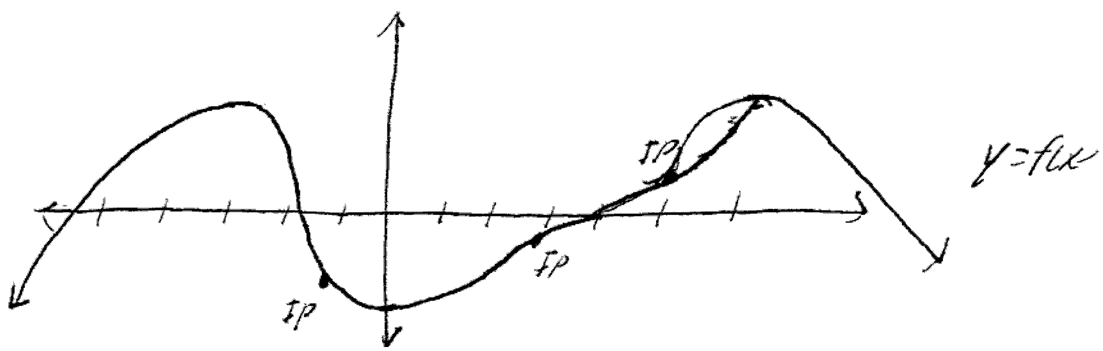
Below is the graph of $f'(x)$!



$y=f'(x)$

$f(x)$ increasing on $(-\infty,-3) \cup (0,4) \cup (4,6)$
decreasing on $(-3,0) \cup (6,\infty)$, critical #'s $-3, 0, 4, 6$
local max at $x=-3$, $x=6$
local min at $x=0$
neither at $x=4$

Concave down $(-\infty,-1) \cup (3,4) \cup (5,\infty)$
concave up $(-1,3) \cup (4,5)$ inflection point $x=-1, 3, 4, 5$



Rmk Moving the x axis up or down would not make this incorrect since $f(x)$ and $f(x)+C$ have same derivative.

Problem 41 Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ w/a local max value of 3 at $x = -2$ and a local min value of 0 at 1.

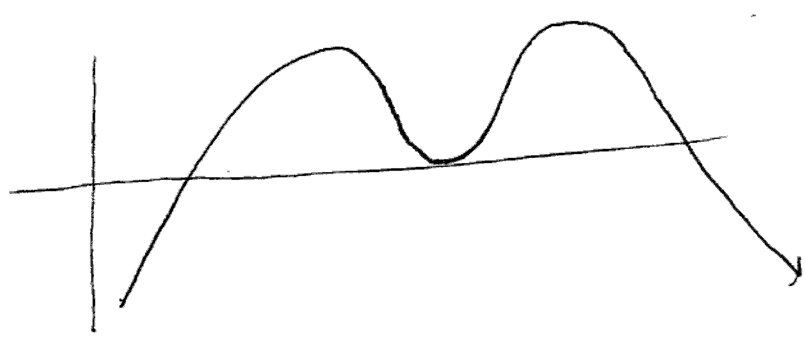
Problem Estimate the x coordinates of inflection points of $f(x)$!

If

a- Curve below is graph of f

b Graph of f'

c- Graph of f''



Problem Find everything and sketch!

$h(x) = x - 2\sqrt{x}$

$f(x) = \dfrac{3x^2}{(x-4)^2}$ $\qquad$ $f(x) = \dfrac{x^2 - 1}{x^3}$

$f(x) = x^4 - 2x^2 + 3$

Problem · Show cubic polynomial always has exactly one inflection pt.

· Show that if cubic has 3 roots, the IP is at the average of the roots