p.168 #2, 9, 10, 31, 32

11/6/06

- Go over quiz
- learn vocabulary

Setting things equal to zero

- if $\frac{a}{b}=0$ then $a=0$
- if $a \cdot b \cdot c = 0$ then $a=0$ or $b=0$ or $c=0$

Example $(x-2)(x+5) + (x-2)(3x-1) = 0$

do not set each term $=0$!

$(x-2)(x+5+3x-1)=0$

$(x-2)(4x+4)=0$

$x-2=0$ or $4x+4=0$

$x=2$ or $x=-1$

Curve sketching

Noncalculus:

- Domain
- x intercepts (set $f(x)=0$ & solve if possible)
- y intercept (plug in $x=0$)
- even/odd function to save time
- periodic? (e.g. $f(x)=\sin x \cos^2 x$
- Horizontal asymptotes
  - * at most 2
  - * look at $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$
- Vertical asymptotes
  - * look for dividing by 0, test points to see if $\lim = \infty$ or $-\infty$ on each side

Calculus

- Find $f'(x)$. Then use to find critical #'s
  - look for points where $f'(x)$ DNE
  - set $f'(x) = 0$
  - test points in intervals for increasing/decreasing
  - 1st deriv test local max/min

- Find $f''(x)$
  - look for points where $f''(x)$ DNE or $= 0$
  - break up # line, test for concave up/down

Sketch graph!!

Example $y = 2x^5 - 5x^2 + 1$

intercepts $(0,1)$ roots too hard! No asymptotes: $D = (-\infty, \infty)$

$y' = 10x^4 - 10x = 10x(x^3 - 1) = 10x(x-1)(x^2+x+1)$

critical #'s $x = 0$, $x = 1$

$f'(x)$: $+$ on $(-\infty, 0)$, $-$ on $(0, 1)$, $+$ on $(1, \infty)$

increasing $(-\infty, 0) \cup (1, \infty)$
decreasing $(0, 1)$

local max at $(0,1)$
local min at $(1,-2)$

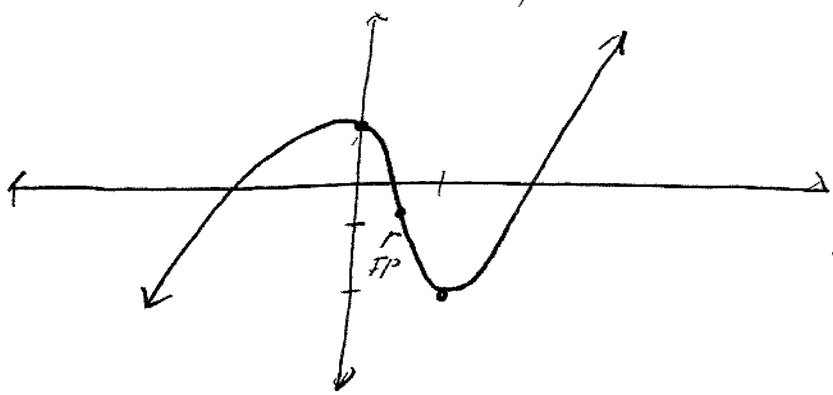
$y'' = 40x^3 - 10$ $\qquad 40x^3 - 10 = 0$
$x^3 = 1/4 \quad x = \frac{1}{\sqrt[3]{4}}$ $\qquad \left(\frac{3}{2}\right)^3 = \frac{27}{8}$ so $\sqrt[3]{4} \approx 1.6$

$-$ on $\left(-\infty, \frac{1}{\sqrt[3]{4}}\right)$, $+$ on $\left(\frac{1}{\sqrt[3]{4}}, \infty\right)$

concave down $\left(-\infty, \frac{1}{\sqrt[3]{4}}\right)$
concave up $\left(\frac{1}{\sqrt[3]{4}}, \infty\right)$

$$f(1/\sqrt[3]{4}) = f(4^{-1/3}) = 2 \cdot 4^{-5/3} - 5 \cdot 4^{-2/3} + 1$$
$$= 4^{-2/3}(2 \cdot 4^{-1} - 5) + 1$$
$$= \frac{1}{\sqrt[3]{16}}(1/2 - 5) + 1$$
$$= -9/10\sqrt[3]{16} + 1$$

I.P. $\left(1/\sqrt[3]{4}, 1 - \frac{9}{10\sqrt[3]{16}}\right)$



Notice $f(1) = -2$ so

21.

$y = \frac{\sqrt{1-x^2}}{x} = \cancel{\sqrt{\frac{1}{x}}}$

vertical asymptote $x = 0$ $\lim_{x\to 0^+} = \infty$ $\lim_{x\to 0^-} = -\infty$

~~$\lim_{x\to\infty}$ = ? when $x>0$ then $x = \sqrt{x^2}$ so $y = \sqrt{\frac{1-x^2}{x^2}} = \sqrt{\frac{1}{}}$~~

<u>Domain</u> $x \neq 0$ and $1 - x^2 \geq 0$ so $\boxed{[-1,0) \cup (0,1] = D}$

x intercepts $(1,0), (-1,0)$

$$y' = \frac{x \cdot \frac{1}{2\sqrt{1-x^2}} \cdot -2x - \sqrt{1-x^2}}{x^2}$$

$$= \frac{-x^2 - (1-x^2)}{x^2\sqrt{1-x^2}} = \frac{-1}{x^2\sqrt{1-x^2}}$$

Decreasing $[-1,0) \cup (0,1]$

$$y'' = \frac{2x\sqrt{1-x^2} + x^2 \cdot \frac{-2x}{2\sqrt{1-x^2}}}{x^4(1-x^2)}$$

$$= \frac{2x(1-x^2) - x^3}{x^4(1-x^2)^{3/2}} = \frac{2x - 3x^3}{x^4(1-x^2)^{3/2}}$$

$$2x - 3x^3 = 0$$

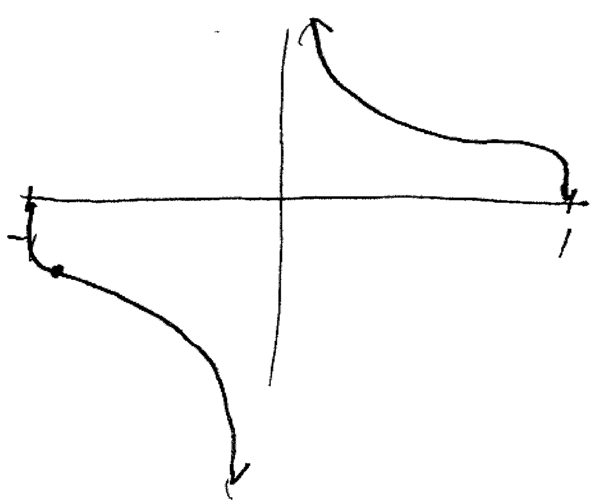
$$x(3x^2 - 2) = 0 \qquad x = 0$$

$$x = \pm\sqrt{2/3}$$

$$\left[\; - \;\underset{-\sqrt{2/3}}{|}\; + \;\underset{0}{|}\; - \;\underset{\sqrt{2/3}}{|}\; + \;\underset{1}{]} \right. \quad f''$$

I.P. $(-\sqrt{2/3}, -1/\sqrt{2})$

$(\sqrt{2/3}, 1/\sqrt{2})$



$$y = \frac{2x}{x^2-4}$$ $\boxed{\text{intercepts } (0,0)}$

$\lim_{x\to\infty} \frac{2/x}{1-4/x^2} = 0$, same for $\lim_{x\to-\infty}$ so $\boxed{y=0 \text{ is H.A.}}$

vertical asymptotes $x=2$ $x=-2$

$\lim_{x\to 2^+} f(x) = \infty$ $\lim_{x\to 2^-} f(x) = -\infty$ $\lim_{x\to -2^+} f(x) = \infty$

$\lim_{x\to -2^-} f(x) = -\infty$

$$y' = \frac{(x^2-4)\cdot 2 - 2x(2x)}{(x^2-4)^2} = \frac{-2x^2-8}{(x^2-4)^2} \leftarrow \text{always} < 0$$

$f(x)$ $\boxed{\text{decreasing } (-\infty,-2) \cup (-2,2) \cup (2,\infty)}$

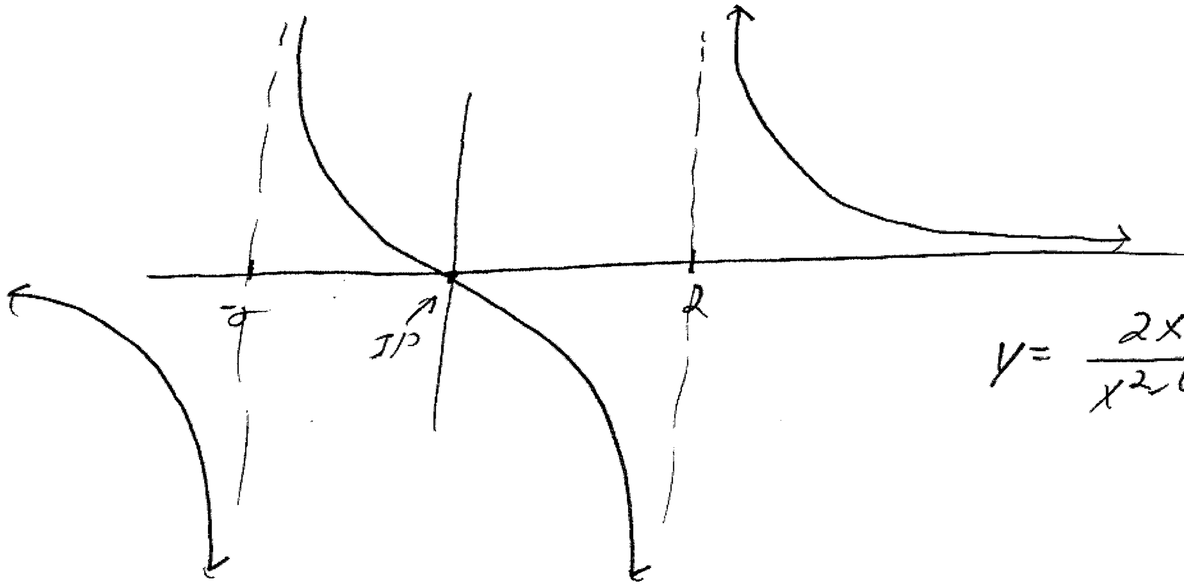
$$y'' = \frac{(x^2-4)^2(-4x) - (-2x^2-8)(2(x^2-4)\cdot 2x)}{(x^2-4)^4}$$

$$= \frac{x(x^2-4)\left[-4(x^2-4) - 4(-2x^2-8)\right)}{(x^2-4)^{\cancel{4}}}$$

$$= \frac{x(4x^2+48)}{(x^2-4)^3}$$

$-$ $+$ $-$ $+$
-2 0 2

concave down $(-\infty,-2) \cup (0,2)$
concave up $(-2,0) \cup (2,\infty)$
I.P. $(0,0)$



$$y = \frac{2x}{x^2-4}$$