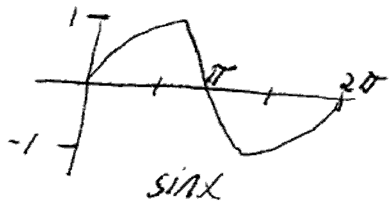
p.176 # 3, 4, 5, 10

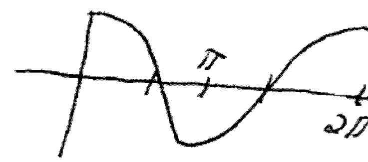
11/7/06 More curve sketching

$$y = \sin x - \tan x = \sin x \left(1 - \frac{1}{\cos x}\right)$$

1. Observe that $f(x) = f(x+2\pi)$ periodic, so we will sketch it on $[0, 2\pi]$.

$\boxed{\text{Intercepts } (0,0) \ (\pi, 0) \ (2\pi, 0)}$ Must know graphs of $\sin x, \cos x$





$y = \cos$

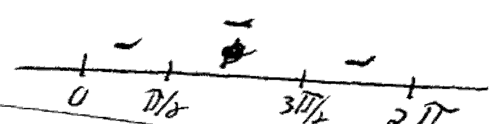
V.A. at $x = \pi/2,\ 3\pi/2$

$\lim_{x \to \pi/2^-} f(x) = -\infty \qquad \lim_{x \to \pi/2^+} f(x) = +\infty$

$\lim_{x \to 3\pi/2^-} f(x) = -\infty \qquad \lim_{x \to 3\pi/2^+} f(x) = \infty$

$$f'(x) = \cos x - \sec^2 x = \cos x - \frac{1}{\cos^2 x} \quad \text{DNE at } x = \pi/2,\ 3\pi/2$$

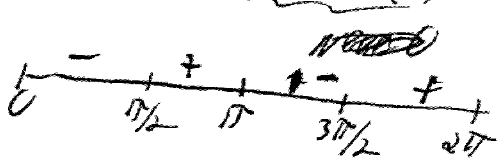
$$0 = \cos x - \frac{1}{\cos^2 x} \Rightarrow \cos^3 x = 1 \Rightarrow x = 0, 2\pi$$



Decreasing on $(0, \pi/2) \cup (\pi/2, 3\pi/2) \cup (3\pi/2, 2\pi)$

$$f''(x) = -\sin x - \frac{-2\cos x(-\sin x)}{\cos^4 x} = -\sin x - \frac{2\sin x}{\cos^3 x} \quad \text{DNE at } \pi/2,\ 3\pi/2$$

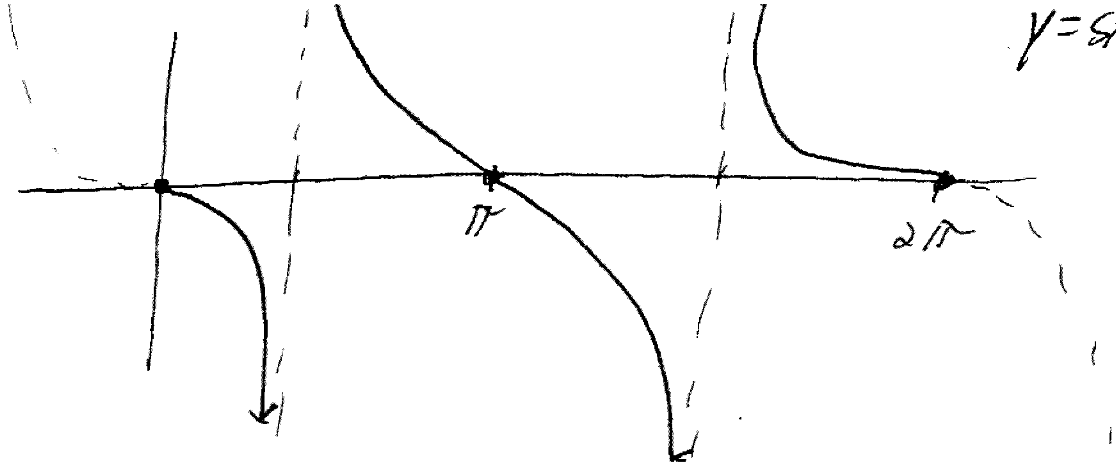
$$0 = -\sin x \left(1 + \frac{1}{\cos^3 x}\right) \quad \text{so } f'' = 0 \text{ at } 0, \pi, 2\pi$$



concave down $(0, \pi/2) \cup (\pi, 3\pi/2)$

concave up $(\pi/2, \pi) \cup (3\pi/2, 2\pi)$

$y = \sin x - \tan x$



Inf. point $(\pi, 0)$
Notice $f'(\pi) = -2$
$f'(0) = f'(2\pi) = 0$

$y = x - 3\sqrt[3]{x}$

Domain $(-\infty, \infty)$
Intercepts $(0,0)$
$(\sqrt[3]{27}, 0)$

$0 = x - 3\sqrt[3]{x}$
$3\sqrt[3]{x} = x$
$27x = x^3 \quad x = \pm\sqrt{27}$

No asymptotes

$y' = 1 - x^{-2/3} = 1 - \frac{1}{\sqrt[3]{x^2}}$ DNE at $x=0$
vertical tangent at $x = 0$

$0 = 1 - \frac{1}{\sqrt[3]{x^2}}$
$\sqrt[3]{x^2} = 1$
$x = \pm 1$ critical #s $-1, 0, 1$

$\overset{+}{\longleftarrow} \underset{-1}{|} \overset{-}{\quad} \underset{0}{|} \overset{-}{\quad} \underset{1}{|} \overset{+}{\longrightarrow}$

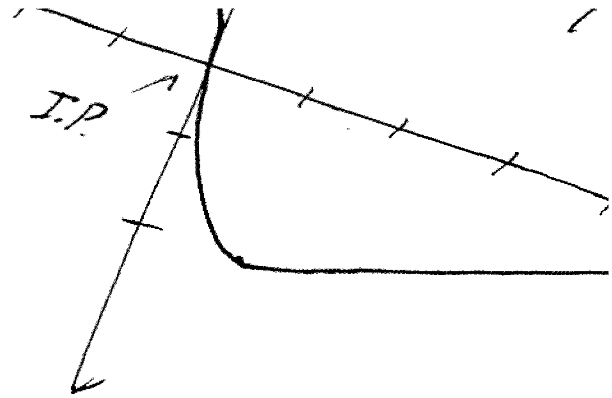
increasing $(-\infty, -1) \cup (1, \infty)$
decreasing $(-1, 0) \cup (0, 1)$

local max $(-1, 2)$
local min $(1, -2)$

$y'' = \frac{2}{3}x^{-5/3} = \frac{2}{3\sqrt[3]{x^5}}$ DNE at $x=0$ $\overset{-}{\longleftarrow} \underset{0}{|} \overset{+}{\longrightarrow}$

concave down $(-\infty, 0)$
concave up $(0, \infty)$

I.P. $(0,0)$



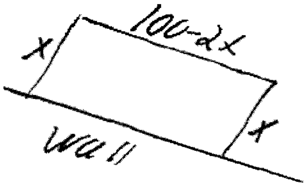
OPTIMIZATION

* Major real world application of calculus *

! Maximize/minimize some quantity as a function of other quantity, subject to some constraints (restrictions)

* Usually want global max/min.

I have 100 ft of fence, make a rectangular pen of maximal area against a wall.



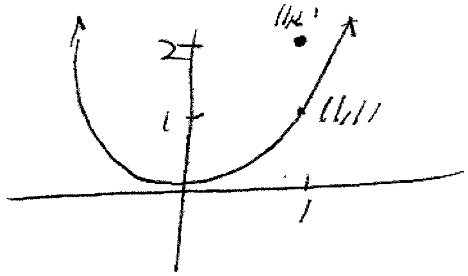
$A(x) = x(100-2x)$
$= 100x - 2x^2 \quad 0 \le x \le 50$

! $A'(x) = 100 - 4x \quad 0 = 100 - 4x \quad x =$

maximum

Problem

Find point on $y=x^2$ closest to $(1,2)$



Let $D(x)$ = distance from $(1,2)$ to point (x,x^2)

$$D(x) = \sqrt{(1-x)^2+(2-x^2)^2} \qquad -\infty < x < \infty$$

$$D'(x) = \frac{1}{2\sqrt{(1-x)^2+(2-x^2)^2}} \cdot \left(2(1-x)(-1)+2(2-x^2)(-2x)\right)$$

Notice $D'(x)$ always exists so set

$$\cancel{2}(1-x)(-1)+\cancel{2}(2-x^2)(-2x)=0$$

$$-1+x-4x+2x^3=0$$

$$2x^3-3x-1=0$$

$$x \approx$$

Global min

Slick method Minimize $D^2(x)$ instead of x!