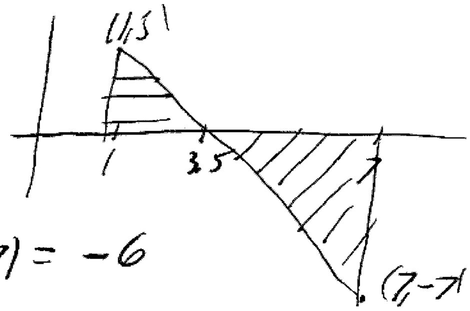
p. 225 #4, 9, 10, 11, 17, 20, 28, 29, 35, 37, 38

11/28/06

Recall $\int_a^b f(x)\,dx$ is definite integral! It gives us "area under curve" where area under x axis is negative, defined via

Example $\int_1^7 -2x+7\,dx$



$= \frac{1}{2}(2.5)(1) - \frac{1}{2}(3.5)(7) = -6$

Properties Assume all integrals below exist.

1. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

2. $\int_a^a f(x)\,dx = 0$

3. $\int_a^b c \cdot dx = (b-a)c$

4. $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

6. If $f(x) \geq 0$ on $[a,b]$ then $\int_a^b f(x)\,dx \geq 0$.

7. If $f(x) \geq g(x)$ on $[a,b]$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

8. If $m \leq f(x) \leq M$ $\forall x \in [a,b]$ then $m \cdot |a-b| \leq \int_a^b f(x)\,dx \leq M\,|a-b|$

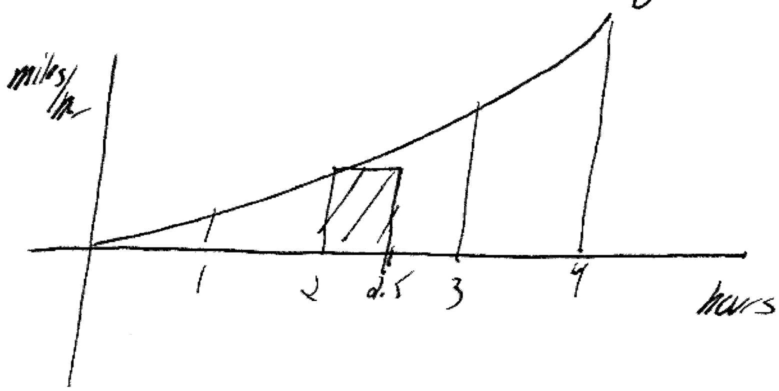
9. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(u)\,du = \cdots$

Evaluating Definite integrals

We seek a better way then using limits of Riemann sums:

Example Suppose $v(t) = s'(t)$. What is $\int_a^b v(t)\,dt$?

Suppose $v(t) = t^2$ mph. Calculate $\int_0^4 t^2\,dt$.



$$L_8 = \frac{1}{2}\cdot v(0) + \frac{1}{2}v(\tfrac{1}{2}) + \frac{1}{2}v(1) + \frac{1}{2}v(\tfrac{3}{2}) + \cdots + \frac{1}{2}v(\tfrac{7}{2})$$

For example $\frac{1}{2}v(2) = \frac{1}{2}\cdot 4 = 2$ miles

* If we stayed at constant speed $v(2)$ from $t=2$ to $t=2.5$
Then we travel 2 miles

** L_8 is an estimate of change in $s(t)$!

*** Integrating $F'(x)$ gives total change in $F(x)$!!

Theorem Suppose $f(x)$ is continuous on $[a,b]$
and $f(x) = F'(x)$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

i.e. $\int_a^b F'(x)\,dx = F(b) - F(a)$

Example $\int_0^1 x^2\,dx$ so $F(x) = \frac{1}{3}x^3$

$= F(1)-F(0) = \frac{1}{3} - 0 = 1/3$

What if $F(x) = \frac{1}{3}x^3 + 3$?

$F(1)-F(0) = 3\frac{1}{3} - 3 = 1/3$

Choice of antiderivative does not matter

Example $\int_0^2 x^3 - 2x^2 - 5\,dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 - 5x \Big|_0^2$

$= \frac{1}{4}\cdot 16 - \frac{2}{3}\cdot 8 - 10 = -11.5$

much faster!!

Example $\int_0^\pi \sin t\,dt = -\cos t\Big|_0^\pi = -\cos\pi - -\cos(0)$

$= 1 + 1 = 2$

Example $\int_{-2}^{2} \sqrt{4-x^2}\,dx = ??$

We know the answer is $\frac{1}{2}\pi\cdot 2^2 = 2\pi$

We don't know an antiderivative

$F(x)$ so $F'(x) = \sqrt{4-x^2}$

Notation

Write $\int f(x)\,dx = F(x)$ for $F'(x) = f(x)$!

indefinite integral!

** $\int_a^b f(x)\,dx$ is a number! $\int f(x)\,dx$ is a function.

Example $\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$

$\int \cos x\,dx = \sin x + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$

etc...

Example

Particle moves w/ velocity $v(t) = t^2 - t - 2$ m/s

~~How far~~ What is total displacement for $t = 0$ to $t = 4$.

** Not total distance **