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11/30/06

Review

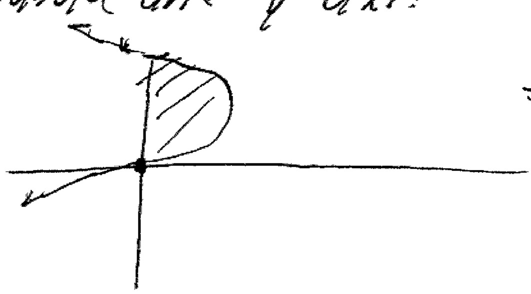
FTOC Part 1 Suppose f is continuous on $[a,b]$ and $F'(x)=f(x)$.
Then
$$\int_a^b f(x)\,dx = F(b)-F(a).$$

Example
$$\int_1^3 \frac{x^2+\sqrt{x}}{x^4}\,dx = \int_1^3 x^{-2}+x^{-7/2}\,dx = \left[-\frac{1}{x} - \frac{2}{5}x^{-5/2}\right]_1^3$$
$$= \left(-\frac{1}{3} - \frac{2}{5}\cdot 3^{-5/2}\right) - \left(-1-2/5\right)$$
$$= -\frac{1}{3} - \frac{2}{5\sqrt{243}} + 7/5$$

Example
$$\int_{-\pi/2}^{\pi/2} \cos y + y^2\,dy = \sin y + \frac{1}{3}y^3 \Big|_{-\pi/2}^{\pi/2}$$
$$= \left(\sin \pi/2 + \frac{1}{3}\left(\frac{\pi}{2}\right)^3\right) - \left(\sin(-\pi/2) + \frac{1}{3}\left(\frac{-\pi}{2}\right)^3\right)$$
$$= 1 + \frac{\pi^3}{24} - \left(-1 - \frac{\pi^3}{24}\right)$$
$$= 2 + \pi^3/12$$

Example

Consider the parabola $x = 4y - y^2$. Find area between parabola and y axis



$$= \int_0^4 4y - y^2\,dy = 2y^2 - \frac{1}{3}y^3 \Big|_0^4$$
$$= \left(32 - \frac{64}{3}\right) - (0)$$
$$= 32/3$$

Indefinite integrals

$$\int x^2+x+3\,dx = \tfrac{1}{3}x^3+\tfrac{1}{2}x^2+3x+C$$

$$\int \sqrt{t}+\sec^2 t\,dt = \tfrac{2}{3}t^{3/2}+\tan t + C$$

Problem Prove that

$$\int \frac{1}{u^2\sqrt{u^2-a^2}}\,du = \frac{\sqrt{u^2-a^2}}{a^2u}+C$$

(Formula # 45 in back)

Proof Take derivative of RHS

$$\frac{a^2u\cdot\frac{2u}{2\sqrt{u^2-a^2}} - a^2\sqrt{u^2-a^2}}{a^4u^2}$$

$$= \frac{a^2u^2-a^2(u^2-a^2)}{a^4u^2\sqrt{u^2-a^2}} = \frac{1}{u^2\sqrt{u^2-a^2}} \checkmark$$

Recall: $\int_a^b F'(x)\,dx = F(b)-F(a)$

integral of rate of change — net change in F

Example #52 If $f(x)$ is slope of trail x miles from start of trail, what does $\int_3^5 f(x)\,dx$ represent.

4.5 "u-substitution"

Recall $\int (x^2+2x)^{10}(2x+2)\,dx = \frac{1}{11}(x^2+2x)^{11} + C$

Why is this true? Chain rule!

Chain Rule Let $u = g(x)$. Then

$$F(x) = f(u) = f(g(x))$$
$$F'(x) = f'(g(x)) \cdot g'(x)$$

Thus we can go in reverse.

Example $\int x\cos(x^2)\,dx = \frac{1}{2}\int 2x\cos(x^2)\,dx$
$= \frac{1}{2}\sin(x^2) + C.$

Formally $u = x^2$

$\frac{du}{dx} = 2x \qquad du = 2x\,dx$

$\int \cos(x^2)x\,dx = \int \cos(u)\cdot\frac{1}{2}\,du$
$= \int \frac{1}{2}\cos u\,du = \frac{1}{2}\sin u + C$
$= \frac{1}{2}\sin(x^2) + C$

Theorem Let $u = g(x)$. Then

$$\cancel{\int f(g(x))} \quad \int f(u)\,du = \int f(g(x))\,g'(x)\,dx$$

u-substitution

1. Goal is to get an antiderivative we know.
2. choose $u = g(x)$ so $du = g'(x)\,dx$
3. Substitute so everything in terms of u
4. Integrate
5. Put x's back

Example

$$\int \sqrt{6x-3}\,dx$$

$$u = 6x-3$$
$$du = 6\,dx$$

$$= \int \frac{1}{6}\sqrt{u}\,du = \frac{1}{6}\cdot\frac{2}{3}u^{3/2} + C$$
$$= \frac{1}{9}(6x-3)^{3/2} + C$$

easy to check!

Example

$$\int \sin 2t\,dt$$

$$u = 2t$$
$$du = 2\,dt$$

$$\int \sin u\ \frac{1}{2}du = -\frac{1}{2}\cos u + C$$
$$= \boxed{-\frac{1}{2}\cos(2t) + C}$$