p. 241 # 11, 17, 23

31, 35, ~~37~~, 39, 41, 43,

53, 55

12/4/06

- Go over quiz

Review $\int \sqrt{-2x+1} + \sin(\tfrac{1}{2}x)\,dx$

$= \int \sqrt{-2x+1}\,dx + \int \sin(\tfrac{1}{2}x)\,dx$

$u = -2x+1$ $\quad$ $u = \tfrac{1}{2}x$

$du = -2dx$ $\quad$ $du = \tfrac{1}{2}dx$

$= \int -\tfrac{1}{2}\sqrt{u}\,du + \int 2\sin u\,du$

$= -\tfrac{1}{3}u^{3/2} - 2\cos u + C$

$= \boxed{-\tfrac{1}{3}(-2x+1)^{3/2} - 2\cos(\tfrac{1}{2}x) + C}$

Definite integrals with u-substitutions

$\int_1^3 (6x+1)^{10}\,dx$ $\quad$ $u = 6x+1$

$du = 6dx$ $\quad$ $x=3 \rightarrow u=19$

$x=1 \quad u=7$

$= \int_7^{19} \tfrac{1}{6}u^{10}\,du = \dfrac{u^{11}}{66}\Big|_7^{19} = \tfrac{1}{66}(19^{11} - 7^{11})$

* change all x's to u's
* change $x=a$, $x=b$ to $u=?$

Ex $\int_0^1 x^2(1+2x^3)^5\,dx$ $\quad u=1+2x^3$ $\quad x=0 \to u=1$

$du=6x^2\,dx$ $\quad x=1 \to u=3$

$$\int_1^3 \frac{1}{6}u^5\,du = \frac{1}{36}u^6\Big|_1^3 = \frac{1}{36}(3^6-3)$$

Ex $\int_0^{\pi/2} \cos^3(t)\sin t\,dt$ $\quad u=\cos t$ $\quad t=0 \quad u=1$

$du=-\sin t\,dt$ $\quad t=\pi/2 \quad u=0$

$$\int_1^0 -u^3\,du = -\frac{u^4}{4}\Big|_1^0 = 0 - -\frac{1}{4} = 1/4$$

Ex Find area under $y=x\sqrt{x^2+1}$ from $x=2$ to $x=5$.

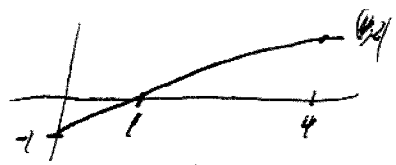
$\int_2^5 x\sqrt{x^2+1}\,dx$ $\quad u=x^2+1$ $\quad x=2 \to u=5$ $\quad x=5 \quad u=26$

$du=2x\,dx$

$\parallel$

$$\int_5^{26} \frac{1}{2}\sqrt{u}\,du = \frac{1}{3}u^{3/2}\Big|_5^{26} = \frac{1}{3}\cdot\left(26^{3/2}-5^{3/2}\right)$$

Ex $\int_0^4 |\sqrt{x}-1|\,dx$



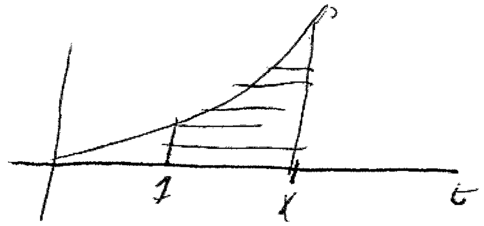
$$= \int_0^1 1-\sqrt{x}\,dx + \int_1^4 \sqrt{x}-1 = \left(x-\frac{2}{3}x^{3/2}\right)\Big|_0^1 + \left(\frac{2}{3}x^{3/2}-x\right)\Big|_1^4$$

$$=(1/3-0) + \left(\frac{16}{3}-4\right)-(-1/3)$$

$$=2$$

Fundamental Theorem of Calculus

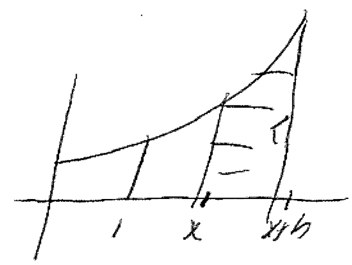
Consider $f(x) = \int_1^x t^2\,dt$. What is this function?



$f(x)$ is the area under $y = t^2$ from $t=1$ to $t=x$

Question: What is $f'(x)$?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$



$$\approx \lim_{h\to 0} \frac{\overset{t^2}{f(x)}\cdot h}{h} = \cancel{f(x)}\; x^2$$

$$f(x) = \int_1^x t^2\,dt = \left.\frac{t^3}{3}\right|_1^x = \frac{x^3}{3} - \frac{1}{3}$$

$$f'(x) = x^2$$

$$** \quad \frac{d}{dx} \int_1^x t^2\,dt = x^2.$$

Fundamental Theorem of Calculus Let f be continuous on $[a,b]$. Then

$$g(x) = \int_a^x f(t)\,dt \qquad a \le x \le b$$

is an antiderivative of f, i.e.

$$g'(x) = f(x)$$

Example Let $g(x) = \int_0^x \sqrt{1+t^2}\,dt$. Find $g'(x)$.

Summary

FTOC Let f be continuous on $[a,b]$. Then

1. If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$
2. $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$

Differentiation & integration are inverses.

Question Why would we ever define a function like

$$g(x) = \int_a^x f(t)\,dt \ ?$$

Example Define

$$\ln(x) = \int_1^x \frac{1}{t}\,dt$$

Then $\ln(x)' = 1/x$, our missing function!