p.234 #1, 3, 7, 9, 15, 17, 25

12/5/06

Review

Fundamental Theorem of calculus

Let $f(x)$ be continuous on $[a,b]$.

1. Let $g(x) = \int_a^x f(t)\,dt$. Then $g'(x) = f(x)$.

2. Suppose F is an antiderivative of f ($F' = f$). Then
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Remarks

#1 says $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$, i.e. integrate then differentiate we are back where we started

#2 says if we start with $F(x)$, diff then integ, we get change in F

Example Find the derivative of $g(x) = \int_0^x \sqrt{1+t}\,dt$

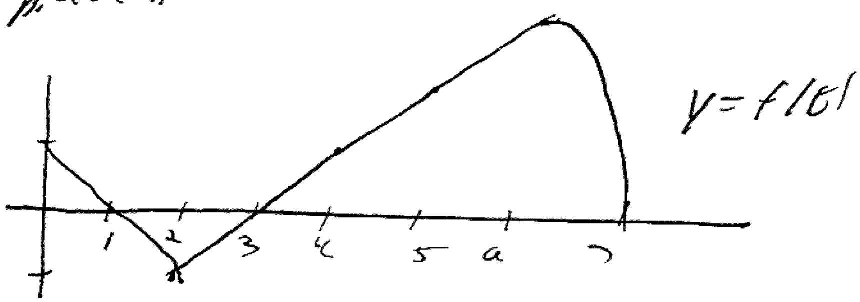
Answer: $\sqrt{1+x}$

Example Define $\ln(x) = \int_1^x \frac{1}{t}\,dt$ for $t > 0$.

Find $\frac{d}{dx}(\ln x)$

Answer: $\frac{1}{x}$

Example p. 234 #2



Let $g(x) = \int_0^x f(t)\,dt$

a. Evaluate $g(0), g(1), g(2), \ldots, g(6)$

b. Estimate $g(7)$

c. Where does g have a max value? min value?

d. Sketch g

Ex #8 $\quad F(x) = \int_x^{10} \tan\theta \, d\theta \quad$ Find $F'(x)$

Ex #14. $\quad y = \int_{\sin x}^{\cos x} (1+v^2)^{10}\,dv \quad$ find $\frac{dy}{dx}$

A: $y = \int_{\sin x}^{0} (1+v^2)^{10}\,dv + \int_0^{\cos x} (1+v^2)^{10}\,dv$

$= -\int_0^{\sin x} (1+v^2)^{10}\,dv + \int_0^{\cos x} (1+v^2)^{10}\,dv$

$u = \cos x$

Let $u = \sin x$

$g(x) \sim g(u) = -\int_0^{\cancel{\sin}\, u} (1+v^2)^{10}\,dv$

$g'(u) = \frac{d}{du}\left(-\int_0^u (1+v^2)^{10}\,dv\right) \cdot \frac{du}{dx}$

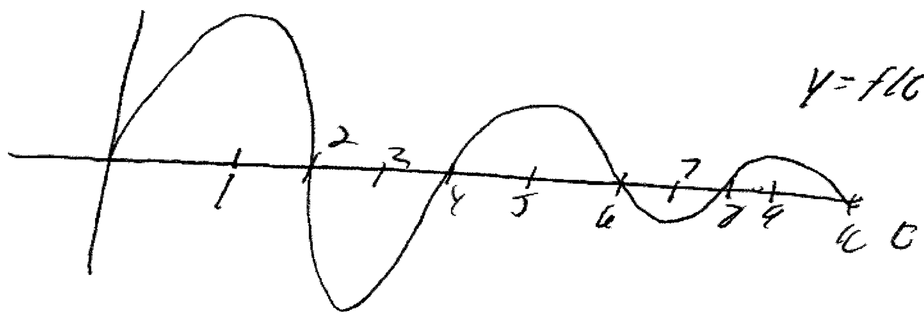
$= -(1+\sin^2 x)^{10} \cdot \cos x$

i.e. $g(u) = \int_0^{\cancel{\cos}\, u} (1+v^2)^{10}\,dv$

This is $g(\cos x)$

Ex. #26

Let $g(x) = \int_0^x f(t)\,dt$



a. Where do local max/mins occur for $g(x)$?

b. Where does $g(x)$ attain absolute max.

c. On what intervals is g concave down?

d. Sketch g?

One last tidbit

Suppose car travels with velocity $v(t) = 1 + t^2$ $t \in [0,5]$.

Problem $\int_0^5 v(t)\,dt$ means what?

A: Total change in distance.

Problem $\int_a^b |v(t)|$ means what?

Problem $\dfrac{\int_0^5 v(t)\,dt}{5}$ means?

Def. The <u>average value</u> of $f(x)$ on $[a,b]$ is

$$f_{ave} = \frac{\int_a^b f(x)\,dx}{b-a}$$

<u>Mean Value Theorem for integrals</u>

f continuous on $[a,b]$ then $\exists\, c \in [a,b]$ so

$$f(c) = f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

<u>Proof</u>

Let $F(x) = \int_a^x f(t)\,dt$. Then

$\exists\, c$ so $\underbrace{\frac{F(b)-F(a)}{b-a}}_{= \frac{\int_a^b f(t)\,dt}{b-a}} = \underbrace{F'(c)}_{= f(c)}$ ✓